DECOMPOSING EVENT-TIMING RANDOMNESS AND EPISTEMIC UNCERTAINTY IN DYNAMIC PRA

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EXTENDED ABSTRACT

Dynamic probabilistic risk assessment (PRA) methods are being developed to explicitly simulate time-dependent system behaviors and stochastic event sequences. These features enable the direct modeling of system dynamics and temporal interactions that are difficult to capture using conventional static PRA methods with event and fault trees. One key advantage of dynamic PRA is its capacity to represent both aleatory and epistemic uncertainties [1] within a unified computational framework. The aleatory uncertainty is stemming from inherent randomness and generally irreducible and the epistemic uncertainty arises from incomplete knowledge and potentially reducible through data collection or model refinement. Therefore, decomposing these two types of uncertainty is essential for risk-informed decision making, as it allows analysts to identify which sources of uncertainty dominate and which can be reduced through additional data or model refinement. This insight, in turn, supports more rational and efficient allocation of resources for risk management actions. This study presents a generalizable framework based on a continuous Markov chain Monte Carlo (CMMC) method [2], a class of dynamic PRA, to quantify and distinguish between epistemic and aleatory uncertainties. A novel metric, the Epistemic-to-Aleatory Ratio (EAR), is proposed to evaluate the relative contribution of epistemic uncertainty to the overall variance in system-level outcomes. The EAR is conceptually adapted from the Gelman–Rubin convergence statistic [3], which compares between-chain and within-chain variances for Markov chain Monte Carlo samplings. To demonstrate the effectiveness of the framework, we conduct a numerical analysis using a holdup tank model [4], which is a simplified reliability model.

Fig. 1 illustrates a generalized simulation workflow for the CMMC method. The core structure consists of a double-loop Monte Carlo sampling procedure. The outermost loop, indexed by $i = 1,...,N$, corresponds to epistemic uncertainty. In this loop, a set of model parameters—such as demand failure probabilities and operational (time-dependent) failure rates—is sampled from their respective probability distributions. These sampled parameters remain fixed throughout the corresponding inner loop. For each outermost-loop sample, the inner loop, indexed by $j = 1,...,M$, represents aleatory uncertainty by generating stochastic realization of the system's behaviors. Each realization simulates the system evolution over time steps ($t = 0, \Delta t,...,T$). At each time step t , system transitions—including component demands, operator actions, or external disturbances—are determined based on the current system state and operational logic. Component failures or state changes are then evaluated probabilistically, using the fixed epistemic parameters. The system state is updated accordingly, and system dynamics are re-evaluated. This process continues until the simulation reaches the final time T or a termination condition is satisfied. After repeating this process for all combinations of N epistemic samples and M aleatory realizations, we obtain a total of $N \times M$ system-level outcomes ψ_{ij} . From these outcomes, we can compute epistemic variance v_{epi} (between-sequence variance) and aleatory variance v_{alea} (within-sequence variance) as:

$$v_{epi} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\overline{\psi}_{i\cdot} - \overline{\psi}_{\cdot\cdot}\right)^2, \text{ where } \overline{\psi}_{i\cdot} = \frac{1}{M}\sum_{j=1}^{M}\psi_{ij},\ \overline{\psi}_{\cdot\cdot} = \frac{1}{N}\sum_{i=1}^{N}\overline{\psi}_{i\cdot}, \tag{1}$$

$$v_{alea} = \frac{1}{N}\sum_{i=1}^{N}{s_i}^2, \text{ where } {s_i}^2 = \frac{1}{M-1}\sum_{j=1}^{M}\left(\psi_{ij} - \overline{\psi}_{i\cdot}\right)^2. \tag{2}$$

Then, EAR is defined as:

$$EAR = \frac{v_{epi}}{\widehat{var}}, \text{ where } \widehat{var} = v_{epi} + \left(1 - \frac{1}{M}\right)v_{alea}. \tag{3}$$

This metric represents the ratio of epistemic variance to the total variance. $EAR > 0.5$ implies that epistemic uncertainty dominates, guiding the need for data acquisition or model refinement. Conversely, an $EAR < 0.5$ suggests that aleatory uncertainty is more influential, and additional resource investment for data acquisition or model refinement should be considered with caution.

To demonstrate the utility of the proposed EAR metric, we apply the framework to the holdup tank model, which simulates water inflow and outflow with probabilistic failures of components (a valve and two pumps) as illustrated in Fig. 2. The simulation procedure follows the CMMC workflow in Fig. 1, and the analytical setup is consistent with our previous study [5], assuming Pump 1 failed-off as the initiating event and dryout—defined as the tank water level dropping to zero—as the endpoint. Epistemic uncertainty is introduced in both demand failure probabilities and time-dependent failure rates. Two cases are analyzed: Case A (base case with a unimodal distribution) and Case B (amplified epistemic uncertainty, represented by a bimodal distribution). Uncertainty propagation to EAR is performed via the CMMC method (Fig. 1). The computed EAR values for Case A and Case B were 0.19 and 0.52, respectively. These results demonstrate that EAR is sensitive to the relative magnitude of epistemic uncertainty and can support resource allocation decisions in data refinement.

In conclusion, this study presents a generalized framework for decomposing two types of uncertainty, epistemic and aleatory uncertainties, in a dynamic PRA using the CMMC method. To quantify the relative contribution of epistemic uncertainty to overall system uncertainty, we propose a novel metric, EAR. The application to a holdup tank model demonstrates how EAR can support the prioritization of epistemic uncertainty reduction in risk-informed decision making. Future work includes extending the proposed metric toward uncertainty importance evaluations.

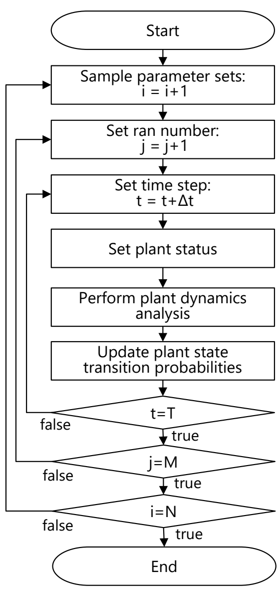

FIGURE 1. CMMC Simulation Workflow

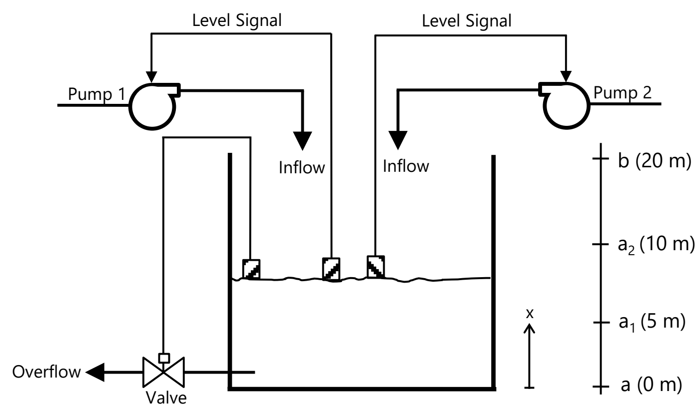

FIGURE 2. Schematic of Holdup Tank Model

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