A MONTE CARLO TECHNIQUE TO PROPAGATE THE UNCERTAINTY OF SEISMICALLY INDUCED JOINT FAILURE PROBABILITY

Shota Soga[1,2*], Eishiro Higo[1], Sunghyon Jang[2], Munemichi Kawaguchi[2], Hiromichi Miura[1]

[1] *Central Research Institute of Electric Power Industry, 2-6-1, Nagasaka, Yokosuka, Kanagawa, 240-0196, Japan, soga@criepi.denken.or.jp**
[2] *Graduate School of Engineering, Hokkaido University, Kita 13-jo, Nishi 8-chome, Kita-ku, Sapporo, Hokkaido 060-8628, Japan, soga@criepi.denken.or.jp**
*Corresponding author

ABSTRACT

Earthquakes are a significant contributor to the risk of nuclear power plants in Japan. An earthquake has the potential to cause simultaneous damage to multiple redundant components, and seismic probabilistic risk assessment quantitatively addresses such a risk. While several approaches have been proposed to estimate the mean value of the seismically induced joint failure probability of multiple components, the propagation of its uncertainty remains an unresolved issue that requires further investigation. This paper addresses this issue and proposes a technique to propagate the uncertainty of the seismically induced joint failure probability of multiple components. First, we review one of the major failure criteria in seismic probabilistic risk assessment. Then, we discuss how to convert this failure criterion into the mean value of the joint failure probability of multiple components. We develop a simple Monte Carlo technique to propagate the uncertainty of the seismic joint failure probability. We provide a numerical demonstration of the proposed technique, showing that the mean value obtained from the constructed distribution of joint failure probability agrees with the expected mean value derived from the multivariate normal model. The proposed algorithm enables uncertainty analysis of the seismically induced joint failure probability of multiple components.

Keywords: Seismically induced joint failure probability, Probability density function, Monte Carlo simulation, Uncertainty analysis, Seismic probabilistic risk assessment

I. INTRODUCTION

Earthquakes are considered one of the major risk contributors to nuclear power plants. For example, several nuclear power plants, such as North Anna Nuclear Power Station, experienced earthquakes resulting in reactor scrams [1]. In the Fukushima Daiichi nuclear power plant accident, the 2011 Tōhoku earthquake and tsunami caused core damage at three units [2]. Thus, operational experience of nuclear power plants demonstrates the need to account for seismic risks in nuclear power plants.

Seismic Probabilistic Risk Assessment (PRA) is a tool that can quantify the seismic risk of nuclear power plants. Seismic PRA estimates risk metrics, such as the core damage frequency of a unit due to an earthquake. There are several challenges in seismic PRA. Seismic fragility analysis (hereinafter referred to as fragility analysis) also presents a challenge in estimating the seismically induced joint failure probability (hereinafter referred to as joint failure probability) of structures, systems, and components (SSCs).

Fragility analysis has two challenging issues. The first issue is the lack of a consensus method to estimate correlation coefficients between seismic responses and between seismic capacities. Researchers have investigated how to estimate these correlation coefficients and their effect on the joint failure probability[3–6]. Given these correlation coefficients, fragility analysts can estimate the mean value of joint failure probability [7]. The multivariate normal (MVN) model is one such model, and there is an efficient algorithm to evaluate this model [8,9]. The second issue is that no method exists to propagate the uncertainty of joint failure probability. To the best of the authors' knowledge, there are no previous studies that investigate how to construct and propagate the uncertainty of joint failure probability, thus representing a significant methodological gap in the field.

In this paper, we address the second issue by proposing a simple Monte Carlo technique for sampling joint failure probabilities. Using the proposed technique enables fragility analysts to construct the uncertainty distribution of a joint failure probability. In Section II, we briefly review a major failure criterion for a single SSC used in seismic PRA. Then, we extend this failure criterion to multiple SSCs. In Section III, we introduce a Monte Carlo technique to sample a seismically induced joint failure probability. In Section IV, we apply the proposed technique to a three-SSC case by propagating their epistemic uncertainties and compare its estimated mean value to that of the MVN model. In Section V, we discuss both advantages and disadvantages of the proposed techniques. Finally, in Section VI, we conclude this study.

The proposed simple technique can also be applied to another failure criterion defined in terms of seismic responses and capacities. Thus, the proposed simple technique is applicable to both failure criteria in seismic PRA.

II. Failure criteria

In Seismic PRA, the most common failure criteria for seismically induced failure of an SSC is given as [10]:

$$A > A_m \epsilon_U \epsilon_R \tag{1}$$

where A is the peak ground acceleration, A_m is the peak ground acceleration at which the mean value of response equals the mean value of capacity, ϵ_U is the random variable for epistemic uncertainty, ϵ_R is the random variable for aleatory uncertainty [7].

It is common to assume that ϵ_U, and ϵ_R follow lognormal distributions expressed as

$$\epsilon_U \sim \mathcal{LN}(0, \beta_U) \tag{2}$$
$$\epsilon_R \sim \mathcal{LN}(0, \beta_R) \tag{3}$$

where β_U, and β_R are the logarithmic standard deviations of epistemic and aleatory uncertainties, respectively.

These failure criteria can be easily extended to simultaneous failures (hereinafter referred to as joint failures) of multiple SSCs. A common failure criterion must be satisfied simultaneously across all SSCs. For example, the failure criterion for joint failure of n SSCs can be expressed in a vector form as

$$\begin{bmatrix} A \\ \vdots \\ A \\ \vdots \\ A \end{bmatrix} > \begin{bmatrix} A_{m,1}\epsilon_{U,1}\epsilon_{R,1} \\ \vdots \\ A_{m,i}\epsilon_{U,i}\epsilon_{R,i} \\ \vdots \\ A_{m,n}\epsilon_{U,n}\epsilon_{R,n} \end{bmatrix}, \tag{4}$$

where $A_{m,i}$ is the median peak acceleration of the ith SSC, and $\epsilon_{U,i}$ and $\epsilon_{R,i}$ are the random variables for epistemic and aleatory uncertainties of the ith SSC, respectively. When Eq. (4) is satisfied, all SSCs are considered to fail.

Epistemic and aleatory uncertainties are assumed to follow a multivariate lognormal distribution expressed as

$$\begin{bmatrix} \epsilon_{U,1} \\ \vdots \\ \epsilon_{U,n} \end{bmatrix} = \boldsymbol{\epsilon_U} \sim \mathcal{MLN}(\mathbf{0}, \boldsymbol{\Sigma_U}) \tag{5}$$
$$\begin{bmatrix} \epsilon_{R,1} \\ \vdots \\ \epsilon_{R,n} \end{bmatrix} = \boldsymbol{\epsilon_R} \sim \mathcal{MLN}(\mathbf{0}, \boldsymbol{\Sigma_R}), \tag{6}$$

where $\mathbf{0}$ is the zero vector, $\boldsymbol{\epsilon_U}$ and $\boldsymbol{\epsilon_R}$ are the vectors of random variables for aleatory and epistemic uncertainties, respectively, and $\boldsymbol{\Sigma_U}$ and $\boldsymbol{\Sigma_R}$ are the covariance matrices of aleatory and epistemic uncertainties. Note that these covariance matrices are expressed in terms of correlation matrices given as $\boldsymbol{\Sigma_U} = \mathrm{diag}(\boldsymbol{\beta_U})\boldsymbol{\rho_U}\mathrm{diag}(\boldsymbol{\beta_U})$ and $\boldsymbol{\Sigma_R} = \mathrm{diag}(\boldsymbol{\beta_R})\boldsymbol{\rho_R}\mathrm{diag}(\boldsymbol{\beta_R})$, where $\boldsymbol{\beta_U}$ and $\boldsymbol{\beta_R}$ are the vectors of epistemic and aleatory uncertainties, respectively and $\boldsymbol{\rho_U}$ and $\boldsymbol{\rho_R}$ are the correlation matrices of epistemic and aleatory uncertainties, respectively.

III. MONTE CARLO TECHNIQUE TO SAMPLE JOINT FAILURE PROBABILITY

In this section, we propose a Monte Carlo technique to sample the seismically induced joint failure probability of n SSCs. First, we develop a Monte Carlo technique for a single SSC, which is then extended to n SSCs.

III.A. Monte Carlo technique for a single SSC

In this section, we propose a Monte Carlo technique for calculating the seismically induced joint failure probability of a single SSC. To demonstrate the proposed technique, we focus on the propagation of epistemic uncertainty under the failure criterion.

Let us take the logarithm of Eq. (1) as $\ln A > \ln A_m + \ln \epsilon_U + \ln \epsilon_R$. The purpose of this transformation is to convert ϵ_U and ϵ_R to normal distributions. Thus, $\ln \epsilon_U$ and $\ln \epsilon_R$ follow normal distributions. This step is not mandatory but simplifies implementation.

In the next step, we sample epistemic uncertainty. Let $\ln \epsilon_{U,j}$ be the jth sampled epistemic uncertainty from a normal distribution. Note that Monte Carlo simulations do not sample the aleatory uncertainty since the aleatory uncertainty is later integrated to define a failure probability. Now, one can transform the failure criterion as follows.

$$\ln A/A_m - \ln \epsilon_{U,j} > \ln \epsilon_R \tag{7}$$

Then, the last step is to convert Eq. (7) into a failure probability. $\ln \epsilon_R$ follows a normal distribution, and thus, one can obtain the probability of satisfying Eq. (7) by taking the integration of $\ln \epsilon_R$ from ∞ to $\ln A/A_m - \ln \epsilon_{U,j}$, which is the cumulative distribution function given as

$$P\left(\ln A/A_m - \ln \epsilon_{U,j} > \ln \epsilon_R\right) = \Phi\left(\frac{\ln A/A_m - \ln \epsilon_{U,j}}{\beta_R}\right), \tag{8}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal distribution. Eq. (8) is interpreted as the conditional failure probability given $\ln \epsilon_{U,j}$. Thus, we can sample failure probability using Eq. (8) and $\ln \epsilon_{U,j}$. Repeating this sampling can construct the probability density of failure probability. In Algorithm 1, we summarize the above Monte Carlo technique.

Input:	1. Peak ground acceleration A 2. Median peak ground acceleration A_m 3. Logarithmic standard deviations β_R and β_U 4. Number of the Monte Carlo trials N
Output:	Sampled failure probabilities $\boldsymbol{P}$
1.	Initialize an empty vector $\boldsymbol{P} = \{\}$
2.	$\{x_1, x_2, \cdots, x_N\} \leftarrow$ Draw N samples from $\mathcal{N}(0, \beta_U)$
3.	For each x_j do
4.	$P_j \leftarrow$ Evaluate Eq. (8) with $\ln \epsilon_{U,j} = x_j$
5.	Append P_j to $\boldsymbol{P}$
6.	Return $\boldsymbol{P}$

Algorithm 1: Monte Carlo procedure to propagate epistemic uncertainty in the seismically induced failure probability of a single SSC

Note that one can sample $\ln \epsilon_U$ using a uniform distribution on [0,1] using the distribution function technique. The cumulative distribution function of $\ln \epsilon_U$ is given by $q = \Phi(\ln \epsilon_U/\beta_U)$. Since q is the cumulative value of $\ln \epsilon_U$, we can consider q follows a uniform distribution on [0,1], and $\ln \epsilon_U$ is expressed as $\ln \epsilon_U = \beta_U \Phi^{-1}(p)$. Then, we can sample the corresponding $\ln \epsilon_U$ by sampling q. Now, let us assume that q_j is the jth sample of q. Now, we get the corresponding jth sample of $\ln \epsilon_U$ as $\ln \epsilon_{U,j} = \beta_U \Phi^{-1}(q_j)$. Thus, Eq. (8) is also given as

$$P\left(\ln A/A_m - \ln \epsilon_{U,j} > \ln \epsilon_R\right) = \Phi\left(\frac{\ln A/A_m - \beta_U \Phi^{-1}(q_j)}{\beta_R}\right). \tag{9}$$

For the single-component case, one can also directly sample failure probability using the equation of the fragility curve. The fragility curve is a function of peak ground acceleration that presents a failure probability [11]. The fragility curve equation is given as

$$F(p, A, A_m, \beta_U, \beta_R) = \Phi\left(\frac{\ln A/A_m + \beta_U \Phi^{-1}(q)}{\beta_R}\right), \tag{10}$$

where q is the analyst's confidence in epistemic uncertainty. Note that one can assume that q follows a uniform distribution. Thus, one can uniformly sample the value of q to propagate the epistemic uncertainty using Eq. (10).

A noticeable difference between Eqs. (9) and (10) is that they have the different signs in front of $\beta_U \Phi^{-1}(p)$. This difference does not affect their probability density. We first show that Eq. (9) can generate the same probability density function as Eq. (10). Figure 1 compares the probability densities constructed by Eqs. (9) and (10) with 10^6 samples.

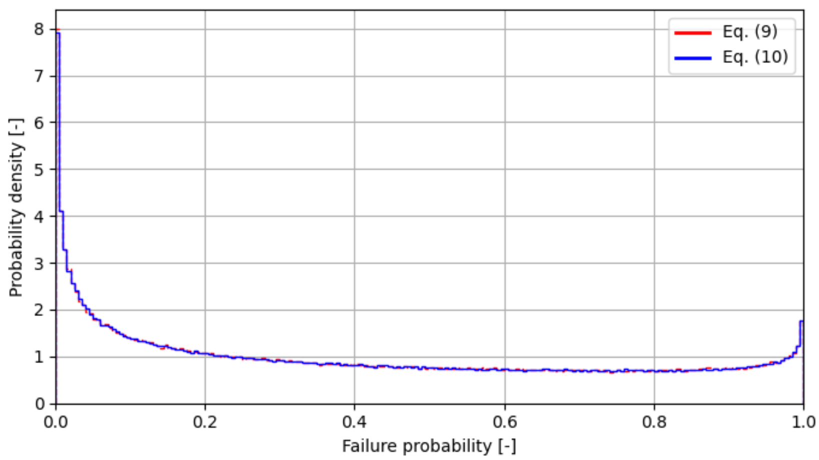

Figure 1. Comparison of the probability densities constructed by Eqs. (9) and (10) with $A = 1.05$, $A_m = 1.1$, $\beta_U = 0.15$, and $\beta_R = 0.12$

Figure 1 shows that Eqs. (9) and (10) yield identical probability densities even though the signs of $\ln \epsilon_{U,j}$ in Eq. (8) and $\beta_U \Phi^{-1}(p)$ in (10) are opposite. This observation can be explained as follows. $\Phi^{-1}(p)$ is symmetric at the origin, implying that $\Phi^{-1}(p)$ and $\Phi^{-1}(-p)$ have the same values for all p, and consequently, $\ln \epsilon_U$ and $-\ln \epsilon_U$ have the same probability density. Figure 1 confirms we can use Eq. (8) to propagate epistemic uncertainty.

Eq. (8) is easily modified to propagate aleatory uncertainty by swapping how the uncertainties are treated. In the above derivation, epistemic uncertainty is sampled, and aleatory uncertainty is integrated to transform the inequality into a failure probability. Instead, we can swap their treatment as the aleatory uncertainty is sampled, and the epistemic uncertainty is integrated to transform the inequality into failure probability. Let $\epsilon_{R,j}$ denote the jth sample of aleatory uncertainty. Then, we can get the following equation:

$$P\left(\ln A/A_m - \ln \epsilon_{R,j} > \ln \epsilon_U \geq -\infty\right) = \Phi\left(\frac{\ln A/A_m - \ln \epsilon_{R,j}}{\beta_U}\right), \tag{11}$$

where $\ln \epsilon_{R,j}$ is the jth sample of aleatory uncertainty. Eqs. (8) and (11) have the same functional form, and therefore, we only need to implement Eq. (8) to evaluate Eq. (11) with different parameters.

III.B. Algorithm for multiple SSCs

In the previous section, we proposed equations to propagate the uncertainty of seismically induced failure probability, with a focus on different types of uncertainties. In this section, we propose equations to propagate the uncertainty of the seismically induced joint failure probability of multiple SSCs. The approach is the same as the single SSC. We first take the logarithm of the failure criterion. Then, we transform a failure criterion into failure probability using the symmetric property and the cumulative distribution function.

Let us consider the failure criterion of Eq. (4) and follow the same steps as the single SSC case. We take the logarithm, sample the epistemic uncertainty of each SSC, and transform the inequality into the failure probability. Let $\epsilon_{U,i,j}$ denote the jth sample value of epistemic uncertainty of the ith SSC. Now, Eq. (4) is transformed as

$$\underbrace{\begin{bmatrix} \ln A \\ \vdots \\ \ln A \\ \vdots \\ \ln A \end{bmatrix}}_{\ln \boldsymbol{A}} - \underbrace{\begin{bmatrix} \ln A_{m,1} \\ \vdots \\ \ln A_{m,i} \\ \vdots \\ \ln A_{m,n} \end{bmatrix}}_{\ln \boldsymbol{A_m}} - \underbrace{\begin{bmatrix} \ln \epsilon_{U,1,j} \\ \vdots \\ \ln \epsilon_{U,i,j} \\ \vdots \\ \ln \epsilon_{U,n,j} \end{bmatrix}}_{\ln \boldsymbol{\epsilon_{U,j}}} > \underbrace{\begin{bmatrix} \ln \epsilon_{R,1} \\ \vdots \\ \ln \epsilon_{R,i} \\ \vdots \\ \ln \epsilon_{R,n} \end{bmatrix}}_{\ln \boldsymbol{\epsilon_R}}. \tag{12}$$

where $\boldsymbol{A}$ is the vector of peak ground accelerations, $\boldsymbol{A_m}$ is the vector of median peak ground accelerations, $\boldsymbol{\epsilon_{U,j}}$ is the vector of jth samples of epistemic uncertainties, and $\boldsymbol{\epsilon_R}$ is the vector of random variables for aleatory uncertainties. Note that $\ln \boldsymbol{\epsilon_R}$ follows a MVN distribution.

Now, let the vector $\boldsymbol{x}$ equal $\ln \boldsymbol{\epsilon_R}$, and we can transform Eq. (12) by taking the cumulative distribution function of an MVN distribution to the failure probability as

$$P(\ln \boldsymbol{A} - \ln \boldsymbol{A_m} - \ln \boldsymbol{\epsilon_{U,j}} > \boldsymbol{x} \geq -\infty) = \int_{-\infty}^{\ln \boldsymbol{A} - \ln \boldsymbol{A_m} - \ln \boldsymbol{\epsilon_{U,j}}} \frac{1}{(2\pi)^{n/2} |\boldsymbol{\Sigma_R}|^{1/2}} \exp\left(-\frac{1}{2} \boldsymbol{x}^{\mathrm{T}} \boldsymbol{\Sigma_R}^{-1} \boldsymbol{x}\right) d\boldsymbol{x}. \tag{13}$$

The right-hand side of Eq. (13) is the formula for the cumulative distribution function of an MVN distribution with zero mean value. Eq. (13) is the conditional failure probability given the jth sampled values of epistemic uncertainties. In the literature, there is a fast Monte Carlo algorithm to evaluate Eq. (12) proposed by Genz and Bretz[1] [9]. Thus, using Eq. (13) to propagate the epistemic uncertainty of failure probability is computationally feasible.

We summarize the steps for propagating epistemic uncertainty using the Monte Carlo sampling technique in Algorithm 1.

Input:	1. Peak ground acceleration A 2. Vector of median peak ground accelerations $\boldsymbol{A_m}$ 3. Covariance matrices $\boldsymbol{\Sigma_R}$ and $\boldsymbol{\Sigma_U}$ 4. Number of the Monte Carlo trials N
Output:	Sampled failure probabilities $\boldsymbol{P}$
1.	Initialize an empty vector $\boldsymbol{P} = \{\}$
2.	$\{\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_N\} \leftarrow$ Draw N samples from $\mathcal{MVN}(0, \boldsymbol{\Sigma_U})$
3.	For each $\boldsymbol{z}_j$ do
4.	$P_j \leftarrow$ Evaluate Eq. (13) with $\ln \boldsymbol{\epsilon_{U,j}} = \boldsymbol{z}_j$ using Genz-Bretz algorithm
5.	Append P_j to $\boldsymbol{P}$
6.	Return $\boldsymbol{P}$

Algorithm 2: Monte Carlo procedure to propagate epistemic uncertainty in seismically induced joint failure probability

Eq. (13) can be easily modified to propagate aleatory uncertainty. By switching the role of epistemic and aleatory uncertainties in the above derivation, we get a similar expression for the conditional failure probability given aleatory uncertainties as

[1] For example, in Python, the multivariate_normal.cdf function in the scipy packages (version 1.14.4) [12] can be used to evaluate Eq. (13).

$$P(\ln \boldsymbol{A} - \ln \boldsymbol{A_m} - \ln \boldsymbol{\epsilon_{R,j}} > \ln \boldsymbol{\epsilon_U}) = \int_{-\infty}^{\ln \boldsymbol{A} - \ln \boldsymbol{A_m} - \ln \boldsymbol{\epsilon_{R,j}}} \frac{1}{(2\pi)^{n/2}|\boldsymbol{\Sigma_U}|^{1/2}} \exp\left(-\frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Sigma_U}^{-1}\boldsymbol{x}\right) d\boldsymbol{x} \tag{14}$$

where $\boldsymbol{x}$ equal $\ln \boldsymbol{\epsilon_U}$, and $\boldsymbol{\epsilon_{R,j}}$ is the vector of the jth samples of aleatory uncertainties. Eq. (14) is the conditional failure probability given the jth sampled values of epistemic uncertainties.

IV. NUMERICAL EXPERIMENT

In this section, we demonstrate the propagation of epistemic uncertainties using Algorithm 2 for a three-component case. We summarize parameter values used in this numerical experiment in Table 1, where $\boldsymbol{\rho_U}$ and $\boldsymbol{\rho_R}$ are correlation matrices of epistemic and aleatory uncertainties, respectively, and $\boldsymbol{\beta_U}$ and $\boldsymbol{\beta_R}$ are the vectors whose ith elements are epistemic and aleatory uncertainties of SSCs, respectively.

Table 1. Input Parameters Employed in the Numerical Demonstration

Parameter	Sample value
A	1.0, 1.1, 1.2
$\boldsymbol{A_m}$	$[1.05 \quad 0.95 \quad 1.00]^{\mathrm{T}}$
$\boldsymbol{\beta_R}$	$[0.12 \quad 0.09 \quad 0.1]$
$\boldsymbol{\beta_U}$	$[0.08 \quad 0.06 \quad 0.07]$
$\boldsymbol{\rho_R}$	$\begin{bmatrix} 1 & 0.3 & 0.2 \\ 0.3 & 1 & 0.4 \\ 0.2 & 0.4 & 1 \end{bmatrix}$
$\boldsymbol{\rho_U}$	$\begin{bmatrix} 1 & 0.7 & 0.6 \\ 0.7 & 1 & 0.8 \\ 0.6 & 0.8 & 1 \end{bmatrix}$
$\boldsymbol{\Sigma_R}$	$\mathrm{diag}(\boldsymbol{\beta_R})\boldsymbol{\rho_R}\mathrm{diag}(\boldsymbol{\beta_R})$
$\boldsymbol{\Sigma_U}$	$\mathrm{diag}(\boldsymbol{\beta_U})\boldsymbol{\rho_U}\mathrm{diag}(\boldsymbol{\beta_U})$

Figure 2 shows the histograms constructed from 10^7 generated samples based on the parameter values shown in Table 1.

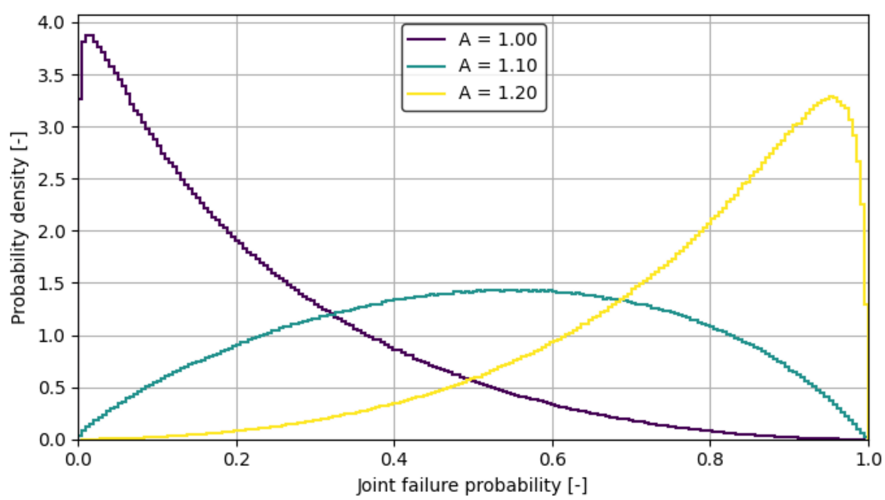

Figure 2. Probability densities constructed from 10^7 samples of joint failure probabilities at 1.0, 1.1, and 1.2

To verify the above result, we evaluate the mean value using the generated joint failure probabilities and comapre it with an MVN model such that

$$\bar{P} = \int_{-\infty}^{\ln \boldsymbol{A} - \ln \boldsymbol{A_m}} \frac{1}{(2\pi)^{n/2} |\boldsymbol{\Sigma_U} + \boldsymbol{\Sigma_R}|^{1/2}} \exp\left(-\frac{1}{2}\boldsymbol{x}^{\mathrm{T}}(\boldsymbol{\Sigma_U} + \boldsymbol{\Sigma_R})^{-\mathbf{1}}\boldsymbol{x}\right) d\boldsymbol{x}. \quad (16)$$

This MVN model is a variation of a model used in the Seismic Safety Margin Research program [8], and that Eq. (16) has a different integration domain. In Table 2, we summarize the comparison of estimated mean values.

Table 2. Comparison of the mean estimations between the proposed model and the MVN model

PGA Value	Proposed technique		MVN model*
	Estimated Mean	Monte Carlo error	
1.0	2.1284e-01	1.4589e-04	2.1282e-01
1.1	5.1483e-01	2.2690e-04	5.1482e-01
1.2	7.7645e-01	2.7865e-04	7.7644e-01

* The reported values are obtained using the internal integration function of the SciPy library, scipy.stats._mvn, under settings that guarantee convergence with absolute and relative errors smaller than 10^{-10}.

Table 2 shows that the proposed technique accurately reproduces the mean estimates of joint failure probabilities derived from the constructed probability densities presented in Figure 2. Thus, this result suggests that the proposed technique can correctly generate the uncertainty distribution of the seismically induced joint failure probability of SSCs.

V. DISCUSSION

We numerically investigated the proposed technique and demonstrated that it can generate the probability density of joint failure probability consistent with the MVN model. One obvious advantage of the proposed technique is its simplicity; fragility analysts can easily implement it. Additionally, the proposed technique does not require any additional parameters to construct the uncertainty of the joint failure probability, provided that fragility analysts have already estimated the mean value of the joint failure probability using the MVN model. This is advantageous because fragility analysts do not have to conduct additional analysis.

Although it is difficult to determine the correlation coefficients between responses and between capacities, it is possible to assume prior distributions for these coefficients and incorporate the constructed probability density of joint failure probability as part of the likelihood function. In doing so, fragility analysts can use a Bayesian updating framework to reduce the uncertainty associated with these correlation coefficients.

As mentioned in the introduction, it is challenging to obtain the correlation coefficients between responses and between capacities which are required parameters for both the MVN model and the proposed technique. Thus, establishing a methodology for estimating these correlation coefficients is also an important direction for future research.

A probability density constructed by the proposed technique inevitably includes a Monte Carlo error. This Monte Carlo error limits the applicability of the proposed technique. For example, it is difficult to obtain accurate gradient information from a constructed probability density. This inaccuracy makes it difficult to use a gradient-based method, such as a Hamiltonian Monte Carlo method. Therefore, establishing an analytical method to derive the probability density function is an important direction for future research.

V. CONCLUSIONS

We proposed a Monte Carlo technique to propagate the uncertainty of seismically induced joint failure probability. The proposed Monte Carlo technique enables us to propagate various uncertainties using the common formula. This feature reduces the implementation cost. We performed numerical experiments using the proposed technique to construct the probability densities at three peak ground acceleration values. Then, we estimated the mean values using the constructed probability densities and compared them to the MVN model. The comparison showed a good agreement between the proposed technique and the MVN model. Therefore, the proposed technique is a good candidate method for propagating the uncertainty of seismically induced joint failure probability.

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