RESOLVING BIASED JUDGMENTS IN RISKY CONTEXT

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ABSTRACT

The paper presents an original symptom-based model and procedure for assessing the cognitive context for a clear and rational interpretation of decision-making process. This heuristic model is applied to explain and assess the subjective understanding and cognition that lead to misjudgments in ambiguous and comparative contexts using the example of Ellsberg's three-color paradox. The idea of overcoming this and other biases is to use the assumption that the wave of symptom recognition by a subject (natural or artificial intelligence) is delayed and disregarded compared to the wave of symptom emergence in an object (socio-technical system) during intuitive conscious information processing. To explain the causes of misjudgments in various erroneous actions, it is necessary to dynamically identify possible trajectories with corresponding probability amplitudes and track their entanglement in the real recognition context. Judgment in an ambiguous comparative context is a non-monotonic wave-like process with successively alternating convex/concave context probability (information entropy), i.e. alternating concave/convex probability of cognition or decision-making as a corresponding sum of varying discrete probability amplitudes of symptom recognition. These probability amplitudes are the result of the interference of objectively existing and subjectively imagined waves of symptoms of the context in a given situation. The groups of symptoms (weighted or not) are modeled by the waves of appearance and recognition of the number of symptoms in the socio-technical system (object and subject in a situation). By adopting and utilizing two overlapping types of additive and subtractive cognitive processes, consisting of different numbers of stages, we can resolve some paradoxes of biased judgments in uncertain and risky contexts and probabilistically comparing them with some known cognitive biases data. The unreliability of judgments in ambiguous and comparative situations is modeled and assessed using the symptom-based procedure of the Performance Evaluation of Teamwork method to qualify and quantify normal or violated contexts.

Keywords: Bias, Judgment, Context, Cognition, Ellsberg three-color Paradox

I. INTRODUCTION

The theory and practice of *human reliability assessment* (HRA) and *human factors* (HF) require extensive use of expert judgment to evaluate *human failure events* (HFE). Rational expert choice in decision-making (DM) requires a deeper understanding of cognitive and judgment processes. Biased judgment or *"rational choice without self-relevant consequences"*[1] is a subjective decision and occurs with some probability when we have unfounded, unfair biases and/or ambiguities due to incorrectly assessed probabilities of consequences (benefits/losses) of a rationally justified decision.

Knight [2] attempted to address these issues by distinguishing between *measurable uncertainty* or *risk* and *unmeasurable uncertainty* or *ambiguity*. Risk is represented in the form of *objectively* known *probabilities* (OP) of *Expected Utility* (EU) theory [3]. Ambiguity is determined by *subjectively* judged *probabilities* (SP) using the *Sure-Thing* (ST) principle in extended *Subjective Expected Utility* (SEU) [4].

Despite its popularity, this theory is not based on in-depth causal, "*conclusive, explanatory models or theories*" of the processes of cognition and decision-making, but rather seeks interesting effects leading to "*a rich set of phenomena*" [5] through paradoxical thinking or fitting approximate formulas. Using sophisticated mathematical models and the borrowing of some

[1] The *"choice entails self-relevant consequence whereas making judgments mostly relies on an impersonal objective evaluation of the situation that has no personal consequences"* [1].

universal ideas from physics, such as quantum mechanics, it attempts to explain nature in order to balance true, pseudo and false randomness.

The gradual way to judge the success of a choice explanation is through empirical testing and validation. But if fitted models provide only approximate and inadequate explanations of effects, they can easily be challenged by experimental results from behavioral economics and psychology.

Savage's ST principle [4] in the extended SEU theory, although partially supported by subsequent real experiments, is largely normatively undermined by a number of paradoxes and effects, such as the thought experiments of Ellsberg [6] and the Machina [7], conjunction and disjunction fallacies, etc. They introduce dissonance and contradictory interpretations, preventing a clear picture of the processes of cognition and decision-making. All of these are used as filters for models of judgment under uncertainty. Very few of the many sophisticated models of rational choice theory survive these filters, and the development of models to explain and resolve these paradoxes and biases continues.

This paper proposes a symptom-based model of cognition and decision-making for a relatively simple explanation of biases and paradoxes associated with the traditional EU model and its adaptations [8], including the resolution of Ellsberg three-color paradox. This is achieved by representing a dynamic array for the temporal context with less than 7 symptom groups with three columns of emerging, recognized, and violated symptoms for each group in a *socio-technical system* and using rules for adding and subtracting the number of symptoms in each column of the array. The columns represented *O x S x V* (*objectively* existing *x subjectively* recognized *x* perceived with *violations*) symptoms in a given situation. Decision makers recognize and minimize context uncertainty (presented as probability) and thus maximize the successful cognition or judgment (presented also as probability).

II. BIASES AND JUDGMENT IN RATIONAL CHOICE THEORY

II.A. Approaches and models to dealing with biased judgments

Many models, methods and approaches have been proposed to generalize biased judgments (normative/axiomatic or non-normative) that consider and accommodate the ambiguity preferences in the Ellsberg and Machina [9] paradoxes [10].

Machina's paradox [7] has been experimentally confirmed by L'Haridon and Placido [11], and many of the real experiments or filters to confirm or undermine these models and theories of rational choice have been extensively studied and described in Machina and Siniscalchi [12].

Such models are: the *Two-Stage (TS) model* in [13], *Choquet EU (CEU)* in [14], *Maxmin Expected Utility (MEU)* in [15], *Bayesian approach based model* in [16], *Cumulative Prospect Theory (CPT)* in [17], *α-Maxmin Expected Utility model (αMEU)* in [18], the *KMM smooth model of ambiguity aversion* in [19], *Variational Preferences model (VP)* in [20], *Vector Expected Utility (VEU)* in [21], *Expected Uncertainty Utility (EUU)* in [22], *Two-Stage Evaluation (TSE)* in [23] among others.

Kitto [24] argues that an appropriate *"mechanism for dealing with such contextual dependency is inbuilt into the quantum formalism itself"* (p. 12). But these quantum-like models do not provide a comprehensible and reasonable *"physical"* (*psychological, neurophysiological, behavioral, biological and/or "psychophysical"*) explanation, but only a simple implicit mathematical landscape of the statistical outcome of making rational or irrational choices. Therefore, we should try to create a causal consistent model of cognition and decision-making that provides a *"physical"* explanation, rather than just speculating with complex mathematics.

The dynamic behavior of the *socio-technical system* (STS) can be described in context as *an object and a subject in a situation*. Dealing with human errors or fallacies require describing and modeling thought processes by measuring, evaluating, and analyzing everything in them that could be inferred, defined, or attributed to contextual situations of the STS that are unrecognizable, undefined, ambiguous, and relatively risky.

The main idea for overcoming these and other paradoxes and fallacies is to use the proposition on the dual symptom-wave nature of the process of cognition and decision-making to justify the fact that at any moment in time the grounds for judgment are based on tracking all possible trajectories of the real context.

II.B. Entropic understanding of cognition and judgment

Explaining complex processes such as cognition and judgment through simple, generally accepted and widely used concepts and ideas would be more understandable and applicable. Therefore, symptoms or stimuli are suitable means that not only provide a comprehensive description of STS information, but also allow for its exchange, filtering and ignoring, i.e., the possibility of organizing a holistic process with a change in STS entropy. Using symptoms alone, information exchange can be achieved quite simply by simultaneously adding new *"task-relevant information"* to STS states and subtracting (*"selectively inhibited at the same time"*) recognized information [25]. Thus, we attempt to provide an entropic understanding and explanation of cognition and judgment through symptom-based context evaluation, which allows modeling quantum-like superposition through trivial arithmetic operations. The symptom-based counting approach is similar to the *"Maya … system of bars and dots … and … rules of counting beans, putting them in, taking them out in different pots to calculate and predict"* [26]. It is conventionally called *quantum-like symptom counting* (QLSC) and serves to describe all alternative states of STS.

Contextual factors and conditions (CFCs) are *symptoms* that dynamically describe the manifestations of all technological, automatic, manual, and thought processes that occur in the STS. Symptoms are signs of the STS in a given situation and are objective, insofar as they reflect the properties and state of the object, and subjective, since the subject perceives, interprets and recognizes them individually or in groups. Symptoms are grouped according to their distinctive features in the relevant domain. Usually 2 to 7 groups of symptoms are sufficient. For example, in such a complex domain as nuclear technology, these could be: *Events*, *Parameters*, *Functions*, *Goals*, *Resources*, *Transitions* and *Actions*. Typically, symptoms are modeled as indistinguishable within a group, but, if possible, they should be weighted both within and across groups for a given domain.

Since each single symptom (1) of the object is recognized by the subject from its absence (0) to its presence (1) in some time interval, it can be trivially assumed that its recognition in the STS can be described quantitatively using probability.

The *Performance Evaluation of Teamwork* (PET) method uses model and procedure for context qualification and quantification to offer a rational and explainable interpretation of the thought processes [27]. The proposed PET model for context of cognition uses ≥ 2 and ≤ 7 groups with 3 columns (*7 x 3*), *V x O x S*, to represent: the number of violated recognition symptoms in column *V* and the shifts between the number of objective symptoms in column *O* (objectively occurring/measured symptoms) and the number of subjective symptoms in column *S* (subjectively recognized symptoms by natural or artificial intelligence) for each group.

Judgment in an ambiguous and comparative context is a wave-like process with successively alternating concave/convex function of *context probability (CP*) or info entropy, and convex/concave function of *cognitive success probability (CSP*). Both functions are evaluated as probabilities, which are the respective sums of discrete *subjective probability amplitudes (SPAs)* of all alternatives (trajectories) of the context. The SPAs are the result of a combination of the shifts between the number of objectively existing vs. subjectively imagined symptoms of the cognitive context. By distinguishing between at least two types of cognition (*additive* and *subtractive*), consisting of different numbers of stages, we can solve many problems of rational choice theory by considering all or representative samples of possible alternatives to contextual progression. The decision maker approximates (minimizes CP) the shifts between the number in the subjective column and the number in the objective column, and when the most favorable context (minimum CP/ maximum CSP) is reached, the judgment is considered the best possible. The addition and subtraction of information is a trivial idea for the processes of cognition and judgment. But only detailed analysis and synthesis through imaginary arithmetic operations can contribute to the understanding of thought processes. But only detailed analysis and synthesis through imaginary arithmetic operations (*addition*, *subtraction*, *multiplication* or *repeated addition of groups of unequal size*) can contribute to understanding the thought processes.

II.C. Synthesis of paradoxes of thinking with symptom-based context

The cognitive context can be defined as a statistical measure of the degree of randomness of the state of a complex STS. The cognitive CP is defined as the ratio of the number of unrecognized to the number of possible available STS states [28]. The uncertainty of the cognitive context has a probabilistic-deterministic nature and can represent a two-dimensional plane of aleatory and epistemic uncertainty. However, due to the objective-subjective nature of the decision-making process, a third dimension of contextual/evaluative uncertainty can be distinguished, characterizing the specific man-machine interaction [29].

Symptoms are stimuli that are meaningful to the operator and have consequences for the behavior, states, and trajectories of the STS. The definition of context also relates it to quantum-like processes and the quantum superposition principle, which states that combinations of symptoms for system states and trajectories *"can be represented by a superposition over different degrees of support for the available choice options"* [30]. CP is a conditional measure of the STS error potential over time. Enumerating all possible and unrecognized accessible states with their *subjective probability amplitudes* (SAP) via information bits, quanta, or waves allows the selection of favorable alternatives.

The PET model and procedure for symptom-based context quantification has already been demonstrated for conjunction and disjunction fallacies [8] and Ellsberg's two-color paradox [31], and is now shown to withstand Ellsberg's three-color filter.

Despite the application of many universal ideas and mathematical models to resolve paradoxes and fallacies under ambiguity and comparison, *"none of the arguments that have been proposed is, at the best our knowledge, considered as conclusive"* [32]. Therefore, the aim of this paper is to propose a PET model and procedure for computing and tracking context to resolve and explain the judgment process.

We intend to apply the acquired insights of the PET method applications to analyze and synthesize the stepwise context and its subjective amplitude probabilities of a cognitive process based on recognition, disregarding and violation of symptoms. In this way, stepwise models of cognition can be demonstrated that are suitable for resolving this and other paradoxes.

III. ELLSBERG'S COLOR PARADOXES

Ellsberg's color problems involve fair sets of items (balls or poker chips), where the first set has *clearly* defined items with one, two or more colors, and the second set has *vague* defined items with two, three or more colored items [33]. From each set containing items with k colors (red, black, yellow, green ...), an item of a certain color can be drawn with the same *Objective Probability*, i.e. the placement of items in the sets provides a uniform distribution of colors.

The *clear* set contains an even or odd number of items (e.g. $n_1=N/k$ items of the 1st, the all 2nd, 3rd, …the k-color items contain $N\text{-}n_1$ items), while the *vague* set contains n_i items of its i-color and ($N\text{-}n_i=\sum n_j\text{-}n_i$) items of its other $k\text{-}1$ colors in an unknown proportion, where $n_j \in [0, N\text{-}N/k]$ and $j=1...k, j\neq i$. If the player's color matches the predicted color, the player wins $€Z$ $(Z>0)$ or a multiple of $€Z$, otherwise she/he wins nothing. Suppose further that the decision maker is a "*consequentialist*" in the sense that she/he is only interested in outcomes; and when it comes to money, she/he prefers more money to less [10].

The results of choosing an item from each set show that the *SP* of drawing (betting) on an item of a certain color are equal if the sets are not compared, and the SPs are different if the sets are compared (drawn from one or the other), since the *SP* of the *"clear" set* is greater than that of the *"vague" set* - $SP_c>SP_v$. In addition, the sum of SPs in the *"vague" set* is different from 1, although they are complementary events to 1 ($\Sigma\, SP_{vk} <1$).

III.A. Stepwise modeling of cognition

Ellsberg [7] argues that the ***SP*** of a personal choice in the presence of uncertainty depends not only on the perceived probability of the event in question but also on its vagueness or ambiguity [33]. He characterizes ambiguity as *"a quality depending on the amount, type, and 'unanimity' of information"* (p.657) or context-awareness as will be interpreted in this paper by the PET method.

In this paper, for *cognitive error probability* (CEP=1-CSP) is used CP but the PET method could use also use other three different "improved" models, which means bringing subjective assessments closer to objective ones, with less CEP based on Rasmussen's *Step Ladder Model* [27]. Some subjective measures of probability may be additive, while others may not be additive [5].

Ambiguity aversion and comparative ignorance can contribute differently to $SP=(1\text{-}CP)$ or $SP=CSP=(1\text{-}CEP)$ and could change in magnitude and sign during the cognitive process. If the cognitive process does not involve comparison (and only *noncomparative games*), then it is trivially additive cumulative cognition and can be justified by the *"cumulative learning theory"* [34]. The cumulative cognition used in the PET method is based on a simple skill of recognizing and adding symptoms based on the previous chronology or previous experience. Thus, only the sequential recognition of the symmetric equal objective probabilities of drawing an item of a certain color from set 1 (*clear*) or set 2 (*vague*) is required, and therefore, for noncomparative games: $SP_1 \approx SP_2$.

The idea is to conduct the supposed process of cognition as possible discrete steps of approximation of the subjectively perceived number of symptoms (items of different colors) to their objective number of several steps (2÷4). The stepwise cognition can occur sequentially, in parallel and jointly with a time difference, resembling Donders' stages [35].

We can summarize the following four types of stepwise cognition, required to enumerate all contextual alternatives. O_i is the column with an objective number of i-color (1) items and S_i is the column with a subjective number of i-color (2) items from clear or vague set:

1. *Stepwise additive cognition with memorizing* (AC_m) in 2 steps - without forgetting the recognized O_i & S_i symptoms (preserving their number). The AC_m scheme is presented in TABLE I for $k=3$ colors.
2. *Stepwise subtractive cognition with disregarding* (SC_d) in 4 steps - with disregard of the already recognized O_i & S_i symptoms (numbers of both is reduced but with shift). The SC_d scheme is presented in TABLE II for $k=3$ colors.
3. *Stepwise heuristic cognition with mixing and/or bypassing* in more than 4 steps - a mixture of the previous two by bypassing the memorizing and/or disregarding of some of the symptoms.
4. *Stepwise violated cognition with memorizing, disregarding, mixing or/and bypassing* in more than 4 steps – recognition of one or more groups of symptoms is difficult due to the occurrence of a violations.

III.B. Three-color Ellsberg paradox

In this section, we consider the well-known Ellsberg three-color paradox [7]. This case is sufficiently representative of the paradoxes and the solutions proposed by the PET method. The logic is applicable to other examples, and the PET solutions were also applied to the conjunction and disjunction fallacy [8] and Ellsberg two-color paradox [31]. In all cases, the procedure is carried out in 3 stages:

TABLE I. Stepwise additive cognition with memorizing **(AC_m)** for ***k=3*** colors.

A3_1	Context Factors and Conditions (items)								
Color/type of symptoms	Red			Black			Yellow		
Step number	VO	O	S	VO	O	S	VO	O	S
0/1	0	N	0	0	2N-n	0	0	n	0
2	0	N	1	0	2N-n	0	0	n	0
A3_2	Context Factors and Conditions (items)								
Color/type of symptoms	Red			Black			Yellow		
Step number	VO	O	S	VO	O	S	VO	O	S
0/1	0	N	0	0	2N-n	0	0	n	0
2	0	N	0	0	2N-n	1	0	n	0
A3_3	Context Factors and Conditions (items)								
Color/type of symptoms	Red			Black			Yellow		
Step number	VO	O	S	VO	O	S	VO	O	S
1	0	N	0	0	2N-n	0	0	n	0
2	0	N	0	0	2N-n	0	0	n	1

TABLE II. Stepwise subtractive cognition with disregarding **(SC_d)** for ***k=3*** colors.

A3_1	Context Factors and Conditions (balls)								
Color/type of symptoms	Red			Black			Yellow		
Step number	VO	O	S	VO	O	S	VO	O	S
0/1	0	N	0	0	2N-n	0	0	n	0
2	0	N	1	0	2N-n	0	0	n	0
3	0	N-1	1	0	2N-n	0	0	n	0
4	0	N-1	0	0	2N-n	0	0	n	0
A3_2	Context Factors and Conditions (balls)								
Color/type of symptoms	Red			Black			Yellow		
Step number	VO	O	S	VO	O	S	VO	O	S
0/1	0	N	0	0	2N-n	0	0	n	0
2	0	N	0	0	2N-n	1	0	n	0
3	0	N	0	0	2N-n-1	1	0	n	0
4	0	N	0	0	2N-n-1	0	0	n	0
A3_3	Context Factors and Conditions (balls)								
Color/type of symptoms	Red			Black			Yellow		
Step number	VO	O	S	VO	O	S	VO	O	S
0/1	0	N	0	0	2N-n	0	0	n	0
2	0	N	0	0	2N-n	0	0	n	1
3	0	N	0	0	2N-n	0	0	n-1	1
4	0	N	0	0	2N-n	0	0	n-1	0

1) a description of the decision-making problem is given, together with tables that represent the situation;
2) the context models used to solve the paradox are given in TABLEs I and II, based on the models of the PET used in the procedure for determining the SP of choice and successful judgment;
3) the obtained PET results are compared with the available statistical results for the problem.

III.B.1. Problem description and formulation

In the classic Ellsberg three-color problem, there is an urn containing balls of three colors. The urn includes two parts with a total of *M* (*90*) balls: The *"clear" part* with *M/3=N/2 (30*) balls is red, and the remaining *"vague" part* with *(2*M)/3=N (60)* balls is black or yellow in unknown proportions. It means that the *'vague' part* contains *n* yellow and *(N-n)* black balls in an unknown proportion, where *n ϵ [0, N]*. One ball will be drawn from the urn. A player is asked to bet on one of the acts *b1*, *b2*, *b3* and *b4* defined in Table III. If the color that the player draws is the same as the one she/he predicted, then the player will win *\$Z (Z>0)*, otherwise she/he wins nothing.

TABLE III. Number of balls, color, and payoff matrix of Ellsberg's three-color problem

bet	'clear' part	'vague' part	
color	*red*	*yellow*	*black*
number of balls	***M/3=N/2 (30)***	***n***	***(2*M)/3-n=N-n***
b1, Gamble 1	***\$Z***	***0***	***0***
b2, Gamble 1	***0***	***\$Z***	***0***
b3, Gamble 2	***\$Z***	***\$Z***	***0***
b4, Gamble 2	***0***	***\$Z***	***\$Z***

Suppose players are offered two games as follows:

- *Gamble 1 (comparative objective-subjective).* Player has to guess the color and bet on red (*b1*) vs. black (*b2*) and then choose a ball from the urn.
- *Gamble 2 (comparative subjective-objective).* Player has to guess the one of two colors - red or yellow (*b3*) vs. black or yellow) (*b4*) and then choose a ball from the urn.

III.B.2. Results and comments of experiments

The TABLE IV presents the probabilities of a decision maker for whom *OPs* are determined by the minimal chance of each event occurring and for whom *SPs* are determined by the *principle of indifference* (*insufficient reason*) [10].

In the experiments conducted by Ellsberg and after him, people prefer to bet on the red (*b1*) over the black (*b2*) ball, and to bet on "black and yellow" (*b4*) in front of "red and yellow" (*b3*) balls. This preference cannot be explained by the *EU* hypothesis. The first choice conforms to the ST principle, but the second choice contradicts this principle. Even more this violates the Savage's *ST* principle, which requires the ordering of *b1* to *b2* to be preserved in *b3* and *b4* (since these two pairs differ only in the payoff when a yellow ball is drawn, which is constant for each pair). Nevertheless, these choices are intuitive: *b1* offers the *Z* prize with an *objective* likelihood $OP_1=1/3$, and *b2* offers the same prize but in an element of the *subjective* partition and ambiguity $SP_2 \approx 1/3$ (black, yellow), if the game is fair. In the same way, *b4* offers the prize with an *objective* likelihood $OP_4=2/3$, whereas *b3* offers the same payoff with *subjective* likelihood $SP_3 \approx 2/3$ on the union of the unambiguous event red and the ambiguous event yellow. Evidence shows that in practice this statement often does not hold.

TABLE IV. Principle results of the Ellsberg three-color problem studies

Gamble	'clear' part: red	ratio	'vague' part: black & yellow
1. comparative *objective-subjective*	$\boldsymbol{OP_1 = 1/3}$	>	$\boldsymbol{SP_2 \approx 1/3}$
2. comparative *subjective-objective*	$\boldsymbol{SP_3 \approx 2/3}$	<	$\boldsymbol{OP_4 = 2/3}$

This pattern of preferences is inconsistent with EU theory because it implies that the vague SP_2 of *'black'* is less than OP_1 of *'red'* ($SP_2 < OP_1 = 1/3$) and the vague SP_3 with unknown proportion between *'red* or *yellow'* balls is less than OP_4 of *'yellow* or *black'* ($SP_3 < OP_4=2/3$). Therefore, the sum of the $SP_2 + SP_3 < 1$ for these outcomes in the decision-making process is not equal to one, although these are complementary events.

Ellsberg found that decision makers generally prefer bets with higher *OPs*, *ceteris paribus* (i.e., other *things being equal*, in those situations where the combined probabilities and payoffs are the same in the urns). That is, he found that people are prone to ambiguity. *"Thus, in both cases the unambiguous bet is preferred to its ambiguous counterpart, a phenomenon called ambiguity aversion by Ellsberg"* [36].

III.B.3. Modeling and resolving the Ellsberg three-color problem

To resolve Ellsberg's three-color paradox, we apply the two models of cognitive context, simulated with the PET context quantification procedure based on TABLE I for *additive cognition with memorizing* and TABLE II for *subtractive cognition with disregarding*.

For the non-comparative games (Gamble 1), the AC_m model is used, and for the comparative games (Gamble 2), the SC_d model. With their help we can obtain the following results shown in TABLE V and TABLE VI.

TABLE V. Subjective probabilities calculated by PET method for the Ellsberg three-color problem.

color	Subjective Probability - memorize		Subjective Probability - disregard			
	SP(AC_m) Step 1	**SP(AC_m) Step 2**	**SP(SC_d) Step 1**	**SP(SC_d) Step 2**	**SP(SC_d) Step 3**	**SP(SC_d) Step 4**
red	0,146606	c	0,146606	0,293211	0,302985	0,151492
black	0,146606	0,293211	0,146606	0,293211	0,301927	0,150964
yellow	0,146606	0,293211	0,146606	0,293211	0,301927	0,150964
SUM	**0,439817**	**0,879633**	**0,439817**	**0,879633**	**0,906839**	**0,453420**

TABLE VI. *Objective and subjective probabilities on Ellsberg's 3-color problem* [10] and PET results for them.

bet	'clear' part	'vague' part		Combined probability
color	*red*	*yellow*	*black*	(OP)
b1, Gamble 1	OP=*1/3*	0	0	**b2 < b1 = 0,333333** = (1/3)
b2, Gamble 1	*0*	$SP_c(AC_m)_2$=0,293211	0	**b1 > b2=0,293211** < (1/3)
b3, Gamble 2	OP=*1/3*	$SP_v(SC_d)_3$=0,302985	0	**b4 < b3 = 0,636318** (≈2/3)
b4, Gamble 2	*(2/3)*	$SP_v(SC_d)_3$=0,302985	$SP_v(SC_d)_3$=0,302985	**b3 > b4 = 0,605970** (≈2/3)

IV. CONCLUSIONS AND INSIGHTS

Modeling, explaining, and resolving Ellsberg's three-color paradox allows us to present a heuristic and rational PET methodology for interpreting many experimental psychological studies of problems, fallacies, and paradoxes involving biased thinking and judgment. Therefore, the theoretical development and validation of the models and the procedure of the PET method will continue in the future with studies of known biases with empirical evidence (Machina's color paradoxes, Ellsberg's multi-color paradox (>3), "framing effects", etc.

Avoiding cognitively biased judgment errors primarily involves limiting the effects of delayed or violated symptom recognition through careful qualification, grouping, and quantification of symptoms.

The PET method, based on explicit qualification and quantification of context using simulator data, can increase the confidence in HRA methods and create useful specific databases of *errors of commission*. This dual practical application of the PET method will be realized by using it as the main method in the system for human error collecting data during regular training of WWER-10000 operator crews at the full-scale simulator of the Kozloduy NPP in Bulgaria.

The inability to complete the process of cognition and judgment in ambiguous and comparative contexts is due to limited, delayed, and stochastic capabilities for encoding information in working memory.

This, in turn, limits subsequent access to this information in long-term memory, where the "individual judgment processors" are stored and used through a unified probabilistic thinking logic.

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