Evaluation of Kass-Steffey adjustment for component reliability uncertainty analysis

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ABSTRACT

Reliability data in the probabilistic safety assessment is analyzed by Bayesian inference with empirical data. In the analysis, prior knowledge distribution is updated using empirical data collected from specific data group of interest. When there is a variability in the data groups, such as plant-to-plant variability, the variability distribution is used for prior knowledge distribution. The variability is represented by hyperparameters, and there are uncertainties not only the reliability parameters but also their hyperparameters. Thus, in the context of Bayesian approach, uncertainties in the reliability parameters and their hyperparameters should be analyzed simultaneously. Hierarchical Bayes assigns uncertainty distributions for both the reliability parameters and their hyperparameters and can derive analytic uncertainty distribution for specific data groups. However, due to its mathematical complexity, the hyperparameters are estimated as point values using the empirical Bayes method. Although the empirical Bayes method has been implemented to estimate the variability distribution, there is an uncertainty in the estimate because it varies with the collected data set. Therefore, a Kass-Steffey adjustment also has been implemented to take account the additional uncertainty in the estimate. However, due to several approximations in the Kass-Steffey adjustment, there exists approximation errors in the estimate compared to analytic solution.
This paper presents the mathematical formula of posterior distribution of hierarchical Bayes for fail-to-run events and evaluates the approximation errors in the empirical Bayes method with Kass-Steffey adjustment. In the mathematical formula, it is shown that the support of hyperparameters should be restricted to make proper distribution. Thus, the empirical Bayes method with Kass-Steffey adjustment is compared to three different restriction conditions.

Keywords: Uncertainty analysis, Empirical Bayes, Kass-Steffey adjustment, Hierarchical Bayes, Reliability data

I. INTRODUCTION

In the probabilistic safety assessment (PSA) for nuclear power plants, risk is quantified as a function of input parameters, such as component failure probabilities and failure rates. These parameters are typically estimated using empirical data and statistical inference because the parameters are not directly observable. Thus, state-of-knowledge uncertainties exist in the estimated parameters, and Bayesian inference is adopted to derive the uncertainty distributions for the parameters. A characteristic of Bayesian inference is using prior knowledge distribution to derive current knowledge distribution. When the number of observed data is sufficient, prior knowledge distribution does not significantly affect current knowledge distribution. However, the effect of prior knowledge distribution is increased when the number of observed data is not sufficient, and therefore, it is important to determine appropriate prior knowledge distributions. When there is plant-to-plant variability in the parameters, empirical Bayes method has been widely applied to estimate prior distributions using maximum likelihood estimation. However, an additional source of uncertainties arises in the estimation because empirical data is used to estimate hyperparameters of prior distribution. To handle these uncertainties, Kass and Steffey proposed an adjustment method that approximate posterior expected values and variances and uses the values to approximate current knowledge distribution with defined uncertainty distribution model [1]. The Kass-Steffey adjustment method has been presented to parameter estimation in the PSA when the empirical Bayes is used to estimate prior distributions [2] and implemented for component reliability analysis [3].

Although the Kass-Steffey adjustment reflects additional uncertainties in the estimated hyperparameters by extending variances of current knowledge distribution, there is a limitation corresponding to approximations of the moments and the uncertainty distribution model. Another approach to take account of uncertainties in the hyperparameters is hierarchical Bayes that assigns uncertainty distributions for the hyperparameters, referred to as hyperprior. In the hierarchical Bayes, both prior distribution for parameters and hyperprior are updated to current knowledge distribution and therefore, analytic current knowledge distribution for parameters given hyperprior can be derived. Since the empirical Bayes method including Kass-Steffey adjustment is an approximation for the hierarchical Bayes when diffuse hyperprior is used, their approximation error should be analyzed to identify uncertainties in the quantification method.

This paper presents current knowledge distribution for component independent failure rates in the context of hierarchical Bayes using diffuse hyperpriors with respect to bounds of hyperparameters and evaluate the empirical Bayes and Kass-Steffey adjustment. A brief description of the empirical Bayes and Kass-Steffey adjustment is described in Section II. In Section III, mathematical formula of current knowledge distribution with the hierarchical Bayes is derived. Section IV compares the resultant distributions using a case study and evaluates the approximation error. A conclusion of this paper is presented in Section V.

II. EMPIRICAL BAYES AND KASS-STEFFEY ADJUSTMENT

In the PSA, a basic assumption is that reliability of components varies among the plants to reflect plant-specific characteristics. Thus, plant-specific estimate for PSA input parameters is performed by Bayesian update of prior distribution using plant-specific empirical data. An additional assumption is that component reliability resembles each other even though they are not identical. In this context, population variability distribution is adopted to represent plant-to-plant variability in the parameters, and it is used for the prior distribution. To derive the population variability distribution, empirical Bayes has been implemented using maximum likelihood estimation. The empirical Bayes is based on two-stage hierarchical model, determining model parameters for the specific plant and the following data generation. In the case of fail-to-run events, the data generation is a Poisson process with constant failure rate as a model parameter, and the failure rate comes from the population variability distribution. Due to mathematical characteristics, conjugate prior is typically used for the population variability distribution model.

Empirical Bayes uses maximum likelihood estimation to estimate the hyperparameters, and the likelihood function is a marginal likelihood function that marginalizes model parameters. Since gamma distribution is a conjugate prior for the Poisson likelihood function, posterior distribution for plant-specific failure rate follows another gamma distribution.

The Empirical Bayes method determines the prior empirically and therefore, there is a source of uncertainty in the estimated hyperparameters depending on the empirical data set. Thus, Kass-Steffey adjustment has been implemented to account for the uncertainty when there are few plants of observation. Kass-Steffey adjustment accounts for the uncertainty using first order approximation for expected value and variance of posterior distribution with mode of hyperparameters. When the hyperprior is a uniform distribution, the mode can be substituted by maximum likelihood estimator, which is the results of the empirical Bayes method. Consequently, variance of posterior distribution is extended, while expected value is equal to the result of simple Bayesian update with the prior distribution derived by the empirical Bayes method. Based on the approximated posterior expected value and variance, the posterior distribution for failure rate is approximated as another gamma distribution using moment matching.

Even though Kass-Steffey adjustment can reflect the additional variance that comes from the uncertainties in the hyperparameters, there are approximation errors not only the moments but also the distribution model. The error should be analyzed to provide appropriate information to decision makers.

III. HIERARCHICAL BAYES

Empirical Bayes is typically considered as a half-Bayesian approach because some parameters of interest are estimated as point values empirically. In contrast, hierarchical Bayes is a fully Bayesian approach that uncertainty distributions are assigned to parameter of interest. In the two-stage hierarchical model, hyperparameters present uncertainties in the population variability and there is typically no prior knowledge for the variability. Thus, diffuse prior is applied for the hyperprior, such as uniform prior distribution. Since the Kass-Steffey adjustment assumes a uniform prior for using the result of empirical Bayes method, uniform hyperprior is assumed in this paper. According to Bayes theorem, posterior distribution for the failure rate and hyperparameters is proportional to the product of likelihood function, prior distribution given hyperparameters, and hyperprior. When the hyperprior follows a uniform distribution, the posterior distribution for the failure rate accounting for hyperparameters can be derived by marginalizing the hyperparameters out.

$$f(\lambda_i|k_i, t_i) \propto \int_0^A \int_0^B \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda_i^{\alpha+k_i-1} e^{-\lambda(\beta+t_i)}\, d\alpha d\beta = \lambda_i^{k_i-2} e^{-\lambda_i t_i} \int_0^A \frac{\gamma(\alpha+1,\lambda_i B)}{\Gamma(\alpha)} d\alpha \quad (1)$$

where A and B are upper bounds of the hyperparameters, and γ is a lower incomplete gamma function.

The posterior distribution for the failure rate in Eq. (1) is proper except for a condition that supports of all the hyperparameters are positive real values. Since the posterior distribution depends on the support of the hyperparameters, the double bound condition and two single bound conditions are used to evaluate the empirical Bayes method and Kass-Steffey adjustment. Although the posterior distribution for failure rate follows gamma distribution in the case of infinite upper bound of β, the posterior distributions for the other cases do not follow gamma distribution. Furthermore, the parameters of posterior distribution have differences compared to the result of Kass-Steffey adjustment even when the upper bound of β is infinite and the posterior distribution for failure rate follows gamma distribution.

IV. CASE STUDY

A case study is applied to evaluate approximation error in the empirical Bayes and Kass-Steffey adjustment method. INL/RPT-23-73954 provides performances of emergency diesel generators (EDGs) collected from 1998-2022 [4]. The empirical Bayes method and Kass-Steffey adjustment are implemented when there is a variability in the collected data group, such as plant-to-plant variability. Since INL/RPT-23-73954 provides yearly failure data for fail-to-run events of EDGs, year-to-year variability is analyzed in this study. Table I shows the failure data for EDG fail-to-run and point estimated failure rate for each year.

TABLE I. EDG fail-to-run data from 1998 to 2022

Year	Failures	Operation time (hour)	Failure rate (/hour)
1998	4	11,285	3.544E-04
1999	1	11,694	8.551E-05
2000	7	13,607	5.144E-04
2001	2	14,162	1.412E-04
2002	7	13,233	5.289E-04
2003	10	11,931	8.381E-04
2004	13	11,733	1.107E-03
2005	14	12,293	1.138E-03
2006	4	11,365	3.519E-04
2007	17	11,458	1.483E-02
2008	20	11,615	1.721E-03
2009	8	11,637	6.874E-04
2010	13	11,299	1.150E-03
2011	21	12,332	1.702E-03
2012	11	7,352	1.496E-03
2013	17	7,913	2.148E-03
2014	17	7,228	2.351E-03
2015	12	7,788	1.540E-03
2016	10	7,584	1.318E-03
2017	22	7,390	2.976E-03
2018	10	7,875	1.269E-03
2019	8	7,389	1.082E-03
2020	6	7,405	8.102E-04
2021	4	7.186	5.566E-04
2022	5	7.321	6.829E-04

Based on the failure data, prior distribution and its hyperparameters can be derived using the empirical Bayes method, maximum likelihood estimation with marginal likelihood function.

When the estimated distribution is used as prior distribution, current knowledge distribution for year-specific EDG failure rate can be derived using Bayesian inference and its variance is adjusted using Kass-Steffey adjustment method to account for uncertainties in the estimated hyperparameters depending on the collected data set. Table II shows uncertainty distributions for year-specific EDG failure rates and their variances with respect to implementation of Kass-Steffey adjustment. Due to the uncertainties of the hyperparameters in the empirical Bayes method, it can be shown that there is an additional variance when Kass-Steffey adjustment is implemented.

TABLE II. Posterior distributions and variances for EDG failure rates

Year	Hyperparameters		Variance	
	Simple EB	EB-KS	Simple EB	EB-KS
1998	(6.874, 13885.9)	(6.749, 13633.5)	3.565E-08	3.631E-08
1999	(3.874, 14294.9)	(3.751, 13839.5)	1.896E-08	1.958E-08
2000	(9.874, 16207.9)	(9.749, 16001.8)	3.759E-08	3.807E-08
2001	(4.874, 16762.9)	(4.750, 16335.7)	1.735E-08	1.780E-08
2002	(9.874, 15833.9)	(9.749, 15632.6)	3.938E-08	3.989E-08
2003	(12.874, 14531.9)	(12.748, 14389.9)	6.096E-08	6.157E-08
2004	(15.874, 14333.9)	(15.748, 14220.1)	7.726E-08	7.788E-08
2005	(16.874, 14893.9)	(16.748, 14782.6)	7.607E-08	7.664E-08
2006	(6.874, 13965.9)	(6.749, 13712.0)	3.524E-08	3.590E-08
2007	(19.874, 14058.9)	(19.748, 13969.5)	1.006E-07	1.012E-07
2008	(22.874, 14215.9)	(22.748, 14137.1)	1.132E-07	1.138E-07
2009	(10.874, 14237.9)	(10.749, 14073.4)	5.364E-08	5.427E-08
2010	(15.874, 13899.9)	(15.748, 13789.5)	8.216E-08	8.282E-08
2011	(23.874, 14932.9)	(23.748, 14853.6)	1.071E-07	1.076E-07
2012	(13.874, 9952.9)	(13.748, 9862.5)	1.401E-07	1.413E-07
2013	(19.874, 10513.9)	(19.747, 10446.7)	1.798E-07	1.809E-07
2014	(19.874, 9828.9)	(19.747, 9766.0)	2.057E-07	2.070E-07
2015	(14.874, 10388.9)	(14.748, 10300.8)	1.378E-07	1.390E-07
2016	(12.874, 10184.9)	(12.748, 10085.3)	1.241E-07	1.253E-07
2017	(24.874, 9990.9)	(24.746, 9939.3)	2.492E-07	2.505E-07
2018	(12.874, 10475.9)	(12.748, 10373.5)	1.173E-07	1.185E-07
2019	(10.874, 9989.9)	(10.749, 9874.5)	1.090E-07	1.102E-07
2020	(8.874, 10005.9)	(8.749, 9864.6)	8.864E-08	8.991E-08
2021	(6.874, 9786.9)	(6.749, 9609.2)	7.177E-08	7.310E-08
2022	(7.874, 9921.9)	(7.749, 9764.3)	7.999E-08	8.128E-08

To evaluate the empirical Bayes method and Kass-Steffey adjustment, hierarchical Bayes is applied for failure rate of each year. When the upper bound of α is finite, the upper bound is assumed to be 10 times of α of average performance distribution to make a sufficient upper bound. Figure 1 shows the posterior distribution for failure rate with respect to the analysis method and support of hyperparameters. Three specific year failure rates are presented as representative cases, the minimum failure rate (2001), the median failure rate (2004), and the maximum failure rate (2018). Even though the actual minimum failure rate occurs in case 1999, the representative minimum failure rate case is designated as 2001, the second lowest, because the posterior distribution becomes an improper distribution when the upper bound of β goes to infinite. It can be shown that the distributions of β infinite support are more conservative compared to those of α infinite support. In the case of 2001, the result distribution of double bounded support is very similar to the result of α infinite support. Consequently, the central tendencies of distributions derived from the double bounded support and the empirical Bayes with Kass-Steffey adjustment decrease, compared to those of the single bound conditions, as the point estimated failure rate increases. The increment and decrement of central tendency are larger when the empirical Bayes with Kass-Steffey adjustment compared to the double bounded support case. The reason for these results is associated with a characteristic of the empirical Bayes method. In the empirical Bayes method, prior distribution is determined based on the empirical data collected from all the data set. Therefore, information of other data groups affects the results of group-specific distribution, year-specific in this case. Due to the information, there is a central tendency in the year-to-year variability, and it attracts failure rates to average performance of pooled data. On the other hand, in this case study, the upper bounds of the hyperparameters are assumed to be 10 times the average performance. It is expected that the central tendency of β infinite distribution will shift in the same direction as increases and decreases in the upper bound of α, while the central tendency of α infinite distribution will shift in the opposite direction as increases and decreases in the upper bound of β.

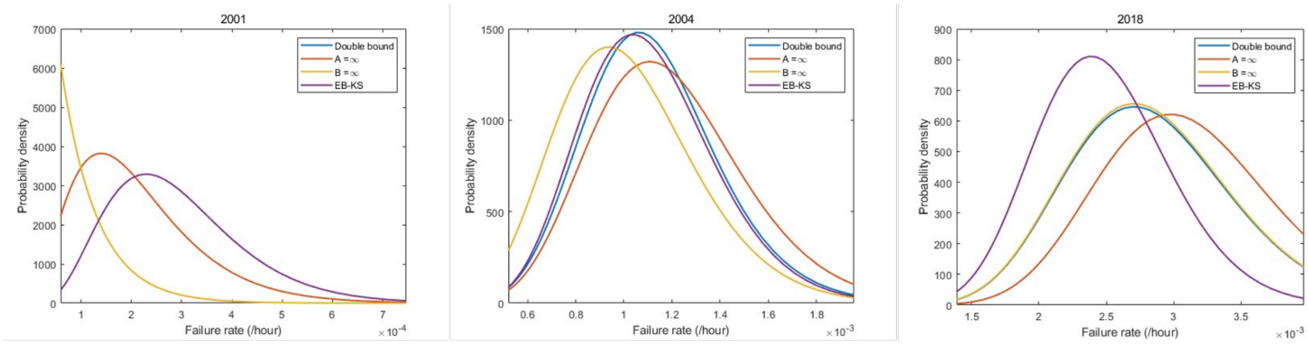

FIGURE 1. Posterior distribution for EDG failure rates

V. CONCLUSIONS

This paper evaluates empirical Bayes and Kass-Steffey adjustment, conventionally implemented in reliability analysis for nuclear power plants using hierarchical Bayes. If the hyperprior is assumed as a uniform distribution, the posterior distribution becomes an improper distribution when the upper bound of support is not restricted. Thus, at least one of the hyperparameters should have a finite upper bound. Two single bound cases and a double bound case are examined, and the results are compared using emergency diesel generator fail-to-run data. It is shown that the double bound case is located between the results of single bound cases. On the other hand, central tendencies of empirical Bayes and Kass-Steffey adjustment decreases compared to the other results from conservative to optimistic as point estimated failure rate of the specific group of interest increases because empirical Bayes contains information from all the data groups and the central tendency of population variability attract the results of empirical Bayes with Kass-Steffey adjustment. Therefore, the empirical Bayes with Kass-Steffey adjustment presents conservative result for low failure rate group and optimistic result for high failure rate group, and the difference is increased as the population variability increases. Thus, approximation error becomes significant when the group of interest has relatively higher failure rate compared to the average performance because the failure rate can be underestimated in this case. In practice, empirical Bayes and Kass-Steffey adjustment remain valuable due to their mathematical simplicity, even though they introduce the approximation errors. However, analysts should account for these errors when there is high population variability in data groups.

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