ANOMALY DETECTION AND SEISMIC DAMAGE INFERENCE OF REACTOR BUILDING USING BAYESIAN INFERENCE

Sunghyon Jang[1], Hideo Machida[2], and Shuhei Matsunaka[2]

[1] *Hokkaido University: Kita-13jo, Nishi-5chome. Kita-ku, Sapporo, Japan*
[2] *TEPCO Systems: 37-28, Eitai 2-chome, Koto-ku, Tokyo, Japan*

ABSTRACT

Early detection of structural damage in nuclear reactor buildings is crucial for safety. In this study, we propose a Bayesian inference framework to detect anomalies, pinpoint damage locations, and quantify damage severity using multi-degree-of-freedom response data. Response analyses under both normal and damaged stiffness conditions yield peak acceleration, velocity, and displacement metrics for each layer. Normalized likelihood functions based on these peak values are used to compute abnormality probabilities via Bayes' theorem. A Bayesian network fuses layer-wise detection results to infer which stiffness element is compromised. Sequential Bayesian updates using shifts in natural periods enable estimation of damage extent (10–40 % stiffness loss).Validation of a three–layer mass–spring–damper model under El-Centro seismic input shows accurate anomaly detection and damage inference across various scenarios. These results demonstrate the promise of Bayesian approaches for structural health monitoring in nuclear reactors.

Keywords: Bayes' theorem, Anomaly detection, Seismic damage, Bayesian inference.

I.BACKGROUND

In the field of nuclear engineering, a multi-degree-of-freedom structural analysis method is used for the assessment of damage to buildings. It is also conceivable that damage to structures caused by earthquakes can occur in forms that are not visible, such as internal damage. If damage to the structure is not detected before it is further damaged by aftershocks, it can lead to a major accident. Therefore, early detection is important for improving the safety of nuclear reactor systems. As such, it is necessary to detect damage by interpreting changes in the behavior of structures using response data from multi-degree-of-freedom structural analysis.

This study aims to construct a model that can estimate the presence and location of damage caused by earthquakes. It does so by conducting multi-degree-of-freedom structural analysis on nuclear reactor buildings and implementing Bayesian inference using response analysis data from both normal conditions and when damage occurs due to earthquakes.

II. ANALITICAL MODEL USING MASS-SPRING-DAMPER STRUCTURE

II. A Analytical Condition

In a three-layer structure, multi-degree-of-freedom response analysis was conducted and validated. Specifically, comparisons were made between normal and abnormal (structural damage) conditions for each layer based on response acceleration, response velocity, and response displacement obtained from the results of the multi-degree-of-freedom response analysis. The multi-degree-of-freedom response analysis involves solving the following differential equation to determine the response acceleration, response velocity, and response displacement[1].

$$[M]\frac{d^2\{x\}}{dt^2} + [C]\frac{d\{x\}}{dt} + [K]\{x\} = -[M]\{I\}\frac{d^2\phi}{dt^2} \quad (1)$$

The natural period T can be calculated from the following formula.

$$det(-\omega^2[M]+[K]) = 0,\ T = \frac{2\pi}{\omega} \quad (2)$$

In this study, structural damage is assumed to involve a decrease in stiffness at each layer, with the degree of damage varying in increments of 10%, ranging from 10% to 40% in the scenarios considered. Table 1 shows analytical conditions[2].

Table 1 The settings for a multi-degree-of-freedom response analysis

Variable Name	Contents	Value	Unit
m_1	Mass of Lv1	4.0×10^5	[kN]
m_2	Mass of Lv2	4.0×10^5	[kN]
m_3	Mass of Lv3	3.0×10^5	[kN]
k_1	Stiffness of Lv1	2.48×10^3	[kN/cm^2]
k_2	Stiffness of Lv2	2.48×10^3	[kN/cm^2]
k_3	Stiffness of Lv3	2.48×10^3	[kN/cm^2]

The model diagram used for the analysis is as follows. In the multi-degree-of-freedom response analysis, the seismic input was the El-Centro NS wave. The waveform is shown below[3].

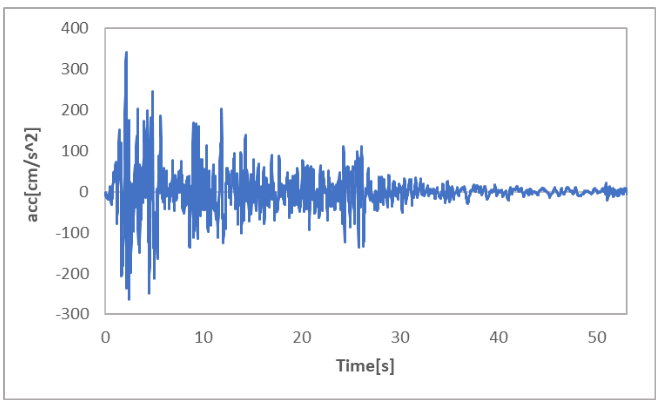

Figure 1 The El-Centro NS wave acceleration

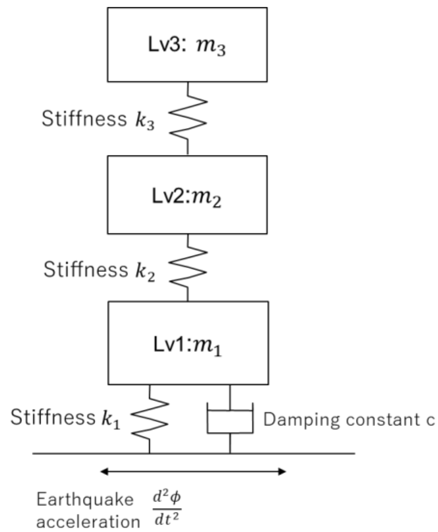

Figure 2 Model overview

The overview of the estimation method used in this study is referred to as the "modular type," which we have named. The specific method is as follows:

1. Conduct a multi-degree-of-freedom response analysis to obtain the response acceleration, velocity, and displacement for each layer, and compare these to normal values to determine if they are normal or abnormal using Bayesian estimation.
2. If abnormalities are detected, use a Bayesian network to identify the affected layer and estimate the location of stiffness damage throughout the system.
3. Employ the natural periods derived from the response analysis in Bayesian estimation to evaluate the extent of damage at the identified locations.

II.B Comparison of Seismic Response Analytical Result between Normal and Abnormal condition

The graphs of the response acceleration, response velocity, and response displacement obtained from the multi-degree-of-freedom response analysis are shown below.

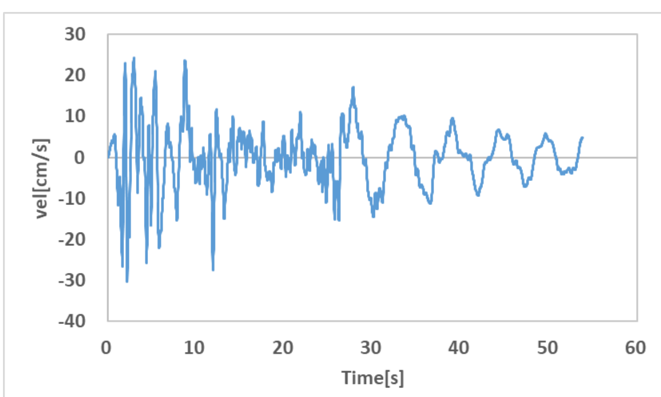

Figure 3 Response velocity at normal condition

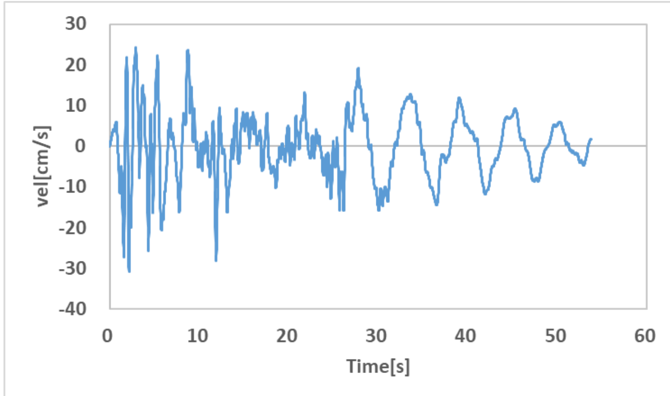

Figure 4 Response velocity at k1 30% damage

Figures 3 and 4 display the response acceleration data for the first layer under normal and abnormal conditions, respectively. It can be observed that there are differences in the magnitude of values and the times at which these values occur between normal and abnormal conditions.

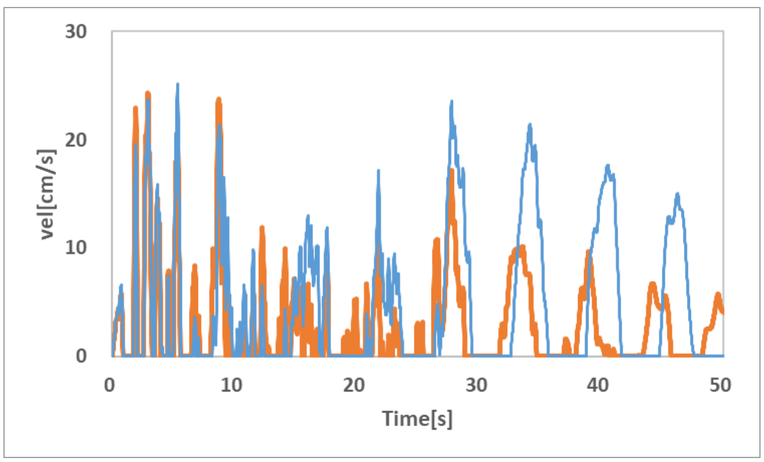

Figure 5 Velocity normal and k1 30% damaged

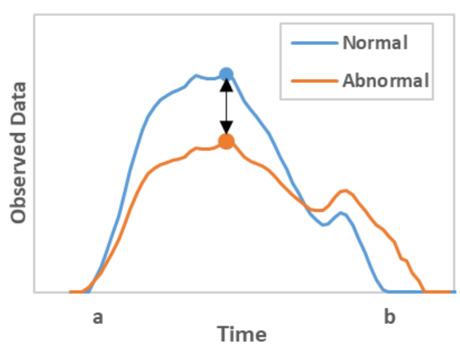

Figure 6 Comparison between normal and abnormal

When defining a positive interval [a, b] in the data during normal conditions, there exists one maximum value within that interval. However, when abnormalities occur, it is thought that there may be shifts in the size of this maximum

value and the timing of their appearance. Focusing on the first layer, the results displaying overlaid data from both abnormal and normal conditions, as shown in Figure 6, are presented below.
In the response acceleration data, it can be confirmed that there are differences in the magnitudes of values and the times at which these values appear between normal and abnormal conditions. It has also been confirmed that similar differences exist in the response velocity and response displacement. Indeed, differences exist between normal and abnormal conditions in terms of the average magnitudes of maximum values and the times at which these maximum values occur. Focusing on these differences, a method for assessing the presence or absence of abnormalities has been investigated.
Here, the comparison between normal and abnormal conditions in terms of the average peak values (peak average) of acceleration, velocity, and displacement data for layers 1 through 3 is shown. The differences were scored and classified into four levels (3, 2, 1, 0). The results are presented in Table 2. This approach allows us to identify which values exhibit significant changes and which exhibit minor changes for each type of damage.

Table 2 Evaluation of the differences compared to normal and abnormal conditions

		Lv1			**Lv2**			**Lv3**		
Peak Average		**Acc**	**Vel**	**Dis**	**Acc**	**Vel**	**Dis**	**Acc**	**Vel**	**Dis**
	k_1	**0**	**3**	**3**	**2**	**3**	**3**	**3**	**2**	**3**
	k_2	**2**	**2**	**2**	**3**	**3**	**3**	**3**	**3**	**3**
	k_3	**2**	**2**	**1**	**2**	**1**	**2**	**0**	**0**	**3**

III. BAYESIAN INFERENCE ON ANOMALY DETECTION

III. A Bayesian Theorem

Bayesian theory is based on Bayes' theorem, which is represented by the following formula:

$$P(S_n|H_n) = \frac{P(S_n)P(H_n|S_n)}{\sum_m P(S_m)P(H_m|S_m)} \tag{3}$$

Posterior Probability, $P(S_n|H_n)$: The probability of hypothesis H_n given the observed event S_n, inferred after evaluating the evidence.
Prior Probability, $P(S_n)$: The initial probability of hypothesis S_n before new evidence, representing our preliminary belief.
Likelihood, $P(H_n|S_n)$: The probability of observing S_n assuming H_n is true, used to update our belief based on new data.
When an earthquake occurs and seismic acceleration data along with the building's response data are obtained, a multi-degree-of-freedom response analysis of the building is performed using the acquired seismic acceleration data. The analysis results are then compared to the building's response data, and this comparison is incorporated into the likelihood function.
The Bayesian formula for evaluating the presence or absence of abnormalities can be expressed as follows:

$$P(Abnormal|Data) = P(Abnormal)\frac{P(Observed\ Data|Abnormal)}{P(Observed\ Data)} \tag{4}$$

The observed data refers to the data obtained from the response analysis results (the average of the interval maximum values). The probability $P(Observed\ Data)$ can be calculated as follows:

$$P(Observed\ Data) = P(Normal)P(Normal|Observed\ Data) + \sum_{i=1}^{N} \mathrm{P}(Abnormal_i)\,\mathrm{P}(Observed\ Data|Abnormal_i) \quad (5)$$

For normal conditions, it is as follows.

$$P(Normal|Observed\ Data) = \mathrm{P}(Normal)\frac{\mathrm{P}(Observed\ Data|Normal)}{\mathrm{P}(Observed\ Data)} \quad (6)$$

Here, the two important likelihoods in Bayesian estimation are defined as follows.
First, a normal distribution is determined using the mean and standard deviation of the maximum value data measured under normal condition. The normal distribution is then adjusted so that the peak density value is 1, and this adjusted distribution is used as the likelihood function. This likelihood function calculates the average of the interval maximum values under certain conditions. The closer this average is to the mean of the interval maximum values under normal conditions, the closer the likelihood value is to 1. Conversely, the further away the average is, the smaller the likelihood value becomes. For the likelihood related to abnormalities, a similar definition is used. The mean of the observed data at each abnormal location (k1, k2, k3) is determined, and then a normal distribution is established using this mean and standard deviation. By adjusting the probability density function so that its maximum value is 1, this function becomes the likelihood function representing the degree of abnormality of the observed data.

III. B Parameters for Bayesian Inference

The likelihood is a necessary value for estimation using Bayesian theory, representing the confidence level of observing event $\boldsymbol{S_n}$ under the condition that cause $\boldsymbol{H_n}$ has occurred. In this study, the likelihood function was created based on a normal distribution as previously described. An example of the created likelihood function is shown below.

$$\boldsymbol{P = \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

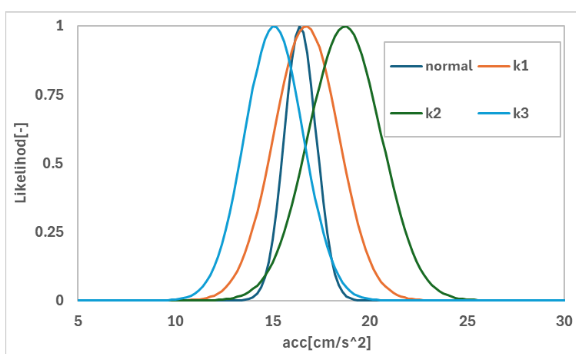

Figure 7 The likelihood function created from the Level 1 response acceleration

Bayesian estimation was conducted on acceleration, velocity, and displacement data using these likelihood functions and equations (3) to (6). Due to the difficulty in obtaining precise prior probabilities $P(Normal)$ and $P(Abormal)$, a uniform distribution was assumed, assigning each a value of 0.5. For abnormalities, given scenarios involving damages k1, k2, and k3, the prior probability of 0.5 was further divided by 3, resulting in equal prior probabilities for each damage scenario.

III.C Bayesian Inference on anomaly detection

Table 3 shows the abnormality degree of the system using the average peak values of acceleration data measured under the assumed damage conditions for each layer. An abnormality degree of 0.5 or higher indicates a correct evaluation result. The evaluation results show that, under certain conditions, the abnormality degree falls below 0.5. This occurs because the data measured under abnormal conditions are similar to the data from normal conditions, leading to the data being mistakenly classified as normal rather than abnormal. This abnormality degree was used as evidence in the Bayesian network explained in the next chapter to evaluate whether each layer shows abnormal values or not.

Table 3 abnormality degree of acceleration data at each level

		Scenario			
		10%	20%	30%	40%
Abnormality	k1	**0.41**	**0.47**	**0.42**	**0.46**
	k2	**0.79**	**0.74**	**0.98**	**1.00**
	k3	**0.43**	**0.89**	**0.76**	**0.76**

IV BAYESIAN NETWORK ANALYSIS ON ANOMALY DETECTION

IV.A Bayesian Network Structure

Bayesian network is a method that integrates the causal relationships between multiple causative events and outcomes, estimating based on conditional probabilities[4].

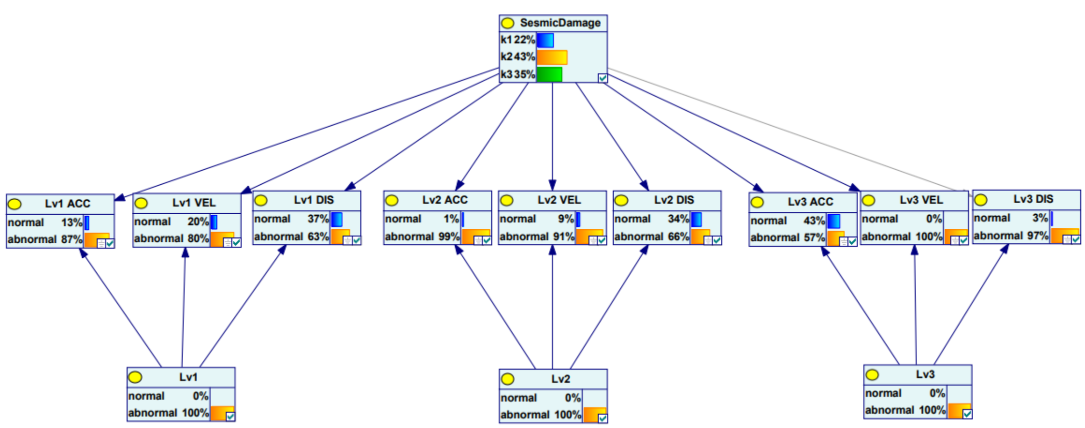

Figure 8 Structure of Bayesian Network

In this study, the difference in abnormality detection involves using acceleration, velocity, and displacement data obtained from each layer for estimation, while damage location estimation uses the acceleration, velocity, and displacement data from all layers for estimation.

IV.B Parameters of Bayesian Network Analysis

In a Bayesian network, conditional probabilities are used to integrate the causal relationships between multiple causative events and outcomes, and estimates are made based on these probabilities. These conditional probabilities are set according to the causal relationships of the outcomes.

Below is the probability table summarizing the conditional probabilities for the first layer response velocity. By incorporating these probabilities into the network structure, it becomes possible to estimate the layer indicating an abnormality.

Table 4 Bayesian Network probability table at Lv1 ACC

Lv1	Normal			Abnormal		
Seismic damage	k1	k2	k3	k1	k2	k3
Normal	0.9	0.9	0.9	0.1	0.25	0.25
Abnormal	0.1	0.1	0.1	0.9	0.75	0.75

This table summarizes the conditional probabilities for the first layer response acceleration. When Lv1 shows normal values and there is no structural damage, it is considered highly probable that k1, k2, and k3 will also show normal values. Therefore, the confidence level for Normal is set at 0.9, and for Abnormal, it is set at 0.1. On the other hand, when Lv1 shows abnormal values, it is considered highly probable that k1, k2, and k3 will also show abnormal values. Therefore, the confidence level for Abnormal is set higher. In Table 4, parameters that are more likely to change have been assigned a higher confidence level for indicating abnormalities. Specifically, the confidence levels for indicating abnormality were set as follows: 0.9 for 3 points, 0.75 for 2 points, 0.6 for 1 point, and 0.5 for 0 points. In the first layer response velocity, as shown in Table 1, k1 has 3 points, and k2 and k3 have 2 points.

IV.C Bayesian Inference on Damage Location

Next, we present the results summarizing the posterior probabilities estimated by integrating acceleration data using the Bayesian network. From these results, it was confirmed that when the true scenario involved k1 and k2 with 10% to 40% damage, and k3 with 20%, 30%, and 40% damage, the posterior probabilities for each layer exceeded 0.5, accurately estimating the presence of abnormalities. However, when the true scenario involved k3 with 10% damage, the posterior probabilities for each layer were below 0.5, failing to accurately estimate the abnormalities.

Table 5 The posterior probabilities estimating the Layer Showing Abnormalities at k1

		True Scenario			
		k1			
		10%	**20%**	**30%**	**40%**
Estimation result	**Lv1**	**0.84**	**0.99**	**0.99**	**0.99**
	Lv2	**0.61**	**0.91**	**0.98**	**1**
	Lv3	**0.96**	**1**	**1**	**1**

Here are the results of using a Bayesian network to estimate damage locations. The table shows the true scenarios and includes the posterior probabilities derived from the results:

Table 6 The Damage Location posterior probabilities estimated by acceleration

		True Scenario			
		k1			
		10%	**20%**	**30%**	**40%**
Estimation result	**k1**	**0.39**	**0.5**	**0.47**	**0.49**
	k2	**0.35**	**0.4**	**0.43**	**0.34**
	k3	**0.26**	**0.1**	**0.1**	**0.1**

When the true scenarios involved damages of 10% to 40% for k1 and k2, and 10% and 20% for k3, it was confirmed that the posterior probabilities for these true scenarios were higher than those for other scenarios, indicating accurate estimations. Furthermore, for true scenarios such as 20% damage in k1, 30% and 40% damage in k2, and 10%

damage in k3, the posterior probabilities were above 0.5, confirming that the estimations were made with high accuracy.

IV.D Bayesian Inference of Degree of Damage

In this section, we estimate the extent of damage in a three-layer structure where an abnormality has been identified and the damage location is known. The values used for this estimation are based on the natural periods, and Bayesian estimation is employed to determine which damage severity scenario is most plausible.

The likelihood functions are created using the same method as the abnormality detection technique, and scenarios of 10% to 40% damage for k1 to k3 are evaluated. Below, we present the likelihood functions for T1 to T3 used for k1 damage, along with an overview of the model used for estimating the damage severity.

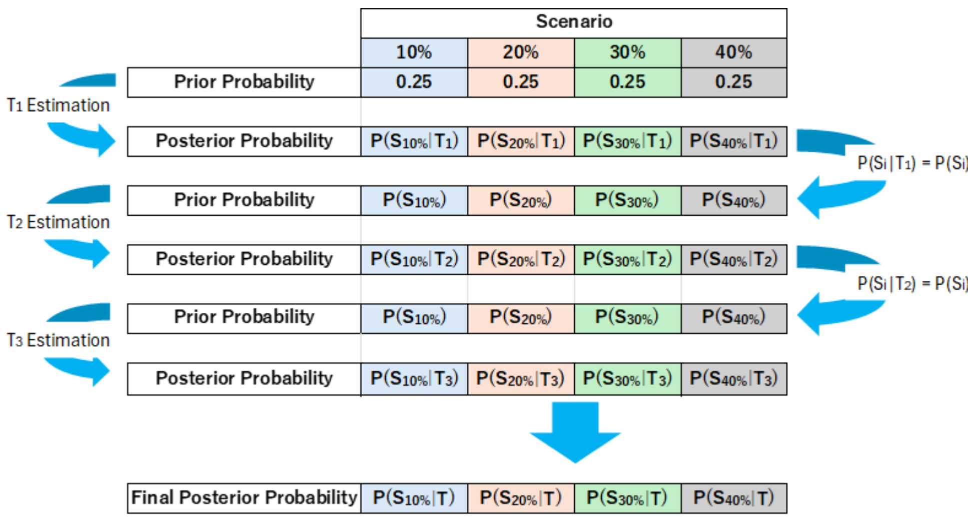

Figure 9 Estimation steps for degree estimation

The estimation steps are as follows:

1. Initial Prior Probability: Set a uniform initial prior probability of 0.25 for each damage scenario (10%, 20%, 30%, 40% for k1, k2, and k3).
2. Sequential Estimations: Perform estimations using the primary (T1), secondary (T2), and tertiary (T3) natural periods, sequentially updating the prior probabilities with the posterior probabilities from the previous step.
3. Final Posterior Probability: Use the posterior probabilities from the tertiary period estimation as the final values for each damage level scenario.

Table 7 Final posterior probability from degree of damage estimation

		True Scenario			
		10%	**20%**	**30%**	**40%**
Final posterior probability	**k1**	**0.77**	**0.69**	**0.80**	**0.91**
	k2	**0.81**	**0.63**	**0.71**	**0.81**
	k3	**0.77**	**0.68**	**0.78**	**0.89**

The above summarizes the results of estimating the degree of damage. As shown in the table, all estimation results exceed 0.5, indicating successful estimation for all damage levels. From these results, it is evident that a method for accurately estimating the damage severity has been established.

IV. CONCLUSIONS

In this study, we examined a Bayesian inference model to estimate the presence, location, and severity of abnormalities in a system using multi-degree-of-freedom response analysis.
For the three-layer model, Bayesian estimation was used to evaluate the presence of abnormalities in each layer when stiffness decreased. The results showed that for Lv1 and Lv2, the model accurately estimated the presence of abnormalities for k1, k2, and k3 under 10% to 40% damage. For Lv3, the model accurately estimated the presence of abnormalities for k1 and k2 under 10% to 40% damage, and for k3 under 30% and 40% damage.
Additionally, a Bayesian network model was constructed to estimate the damage location when abnormalities were detected in each layer. The results showed that the model accurately estimated the damage location for k1 and k2 under 10% to 40% damage scenarios. For k3, the model accurately estimated the damage location under 10% and 20% damage scenarios. Furthermore, Bayesian estimation using the natural periods was conducted to estimate the damage severity. It was confirmed that the model accurately estimated the damage severity for k1, k2, and k3 under all scenarios from 10% to 40%.
The evaluation results indicated some errors in the estimation for Lv3 k3 under 10% and 20% damage in abnormal detection, and for Lv3 k3 under 30% and 40% damage in damage location estimation. To improve this, it is necessary to develop estimation methods that can handle smaller damage levels.
In the future, to achieve more accurate estimations, we will explore and implement the use of machine learning to define the likelihood functions.

Reference

[1] "Seismic Design for the Emergency Response Facilities within the Reactor Buildings of Units 5, 6, and 7 at Kashiwazaki-Kariwa Nuclear Power Station"Available at: TEPCO - Seismic Design (Accessed: 2021)
[2] Kosei Kawamata, Sunghyon Jang, Evaluation on Seismic damage Inference for mass-spring damper structure using Bayesian Theorem, 2N06, Proceedings of AESJ 2023 Fall meeting, Sep.6th-8th, Nagoya, Japan (2023)
[3] The building of Center of Japan Database, El centro earthquake(https://www.bcj.or.jp/download/wave/).
[4] Tu Guang Tan, Sunghyon Jang, Akira Yamaguchi, A novel method for risk-informed decision-making under non-ideal Instrumentation and Control conditions through the application of Bayes' Theorem, Reliability Engineering & System Safety, Vol. 188, PP. 463-472, 2019

REFERENCES

[1] M. P. BROWN and K. AUSTIN, *Title of Book*, pp. 25–30, J. SMITH, Ed., Publisher Name, Publisher City, Publisher State (2004).
[2] M. P. BROWN and K. AUSTIN, "With or Without Title of Paper," *Title of Journal*, **36**, 102 (2004).
[3] R. T. WANG, "Title of Paper," *Proc. Title*, Location of Meeting, Date of Meeting, Vol. No., p. No., Publisher of Proceedings (2004).
[4] "ASRAM2025," https://aseannpsr.com/asram2025/ (2015).